

STA 5364, Report 1

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Report 1.3

To find the survival function of a Weibull distribution, we must find the CDF of a Weibull random variable, whose probability density function can be written as

$$f(x | \alpha, \lambda) = \alpha \lambda x^{\alpha-1} e^{-\lambda x^\alpha}, \quad \alpha > 0, \lambda > 0.$$

So then

$$\begin{aligned} F_X(x) &= \int_0^x \alpha \lambda t^{\alpha-1} e^{-\lambda t^\alpha} dt \\ &= \lambda \int_0^x \alpha t^{\alpha-1} e^{-\lambda t^\alpha} dt \end{aligned}$$

Letting $u = t^\alpha$ and $du = \alpha t^{\alpha-1} dt$, we have that

$$\begin{aligned} F_X(x) &= \lambda \int_0^{x^\alpha} e^{-\lambda u} du \\ &= [-e^{-\lambda u}]_0^{x^\alpha} \\ &= 1 - e^{-\lambda x^\alpha}. \end{aligned}$$

Finally, since $S(x) = 1 - F(x)$, we have that $S(x) = 1 - (1 - e^{-\lambda x^\alpha}) = e^{-\lambda x^\alpha}$.