STA 5364, Report 1.30

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Let X denote a lifetime random variable with a lognormal distributin with scale and shape parameters μ and σ respectively. Thus, $Y = \ln(X)$ has a $\mathcal{N}(\mu, \sigma^2)$ density. Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the PDF and CDF respectively, or a standard normal. Using truncation and censoring notation from the last example, write the likelihood for the transformed observations under left truncation and right censoring.

Solution

From report 1.29, we have that the general likelihood function for left-truncated and right-censored data is

$$L(\cdot|\mathbf{x}) \propto \prod_{i \in \bar{\mathcal{T}}_L} [f(x_i)]^{\delta_i}[1 - F(y_i)]^{1-\delta_i} \prod_{i \in \mathcal{T}_L} \left[\frac{f(x_i)}{1 - F(Y_{L_i})}\right]^{\delta_i} \left[\frac{1 - F(y_i)}{1 - F(Y_{Li})}\right]^{1-\delta_i},$$

where $\delta_i = 0$ if the observation is right censored and 1 if it is not. Therefore the full likelihood for observations is given by:

$$\begin{aligned}\ln L(\mu, \sigma|\mathbf{y}, \boldsymbol{\delta}, \boldsymbol{\nu}) = &\sum_{i \in \bar{\mathcal{T}}_L} \left[\delta_i \phi(y_i) + (1 - \delta_i)\left(1 - \Phi\left(\frac{y_i - \mu}{\sigma}\right)\right)\right] \\ &+ \sum_{i \in \mathcal{T}_L} \left[\delta_i \left(\frac{\phi\left(\frac{y_i - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{Y_L - \mu}{\sigma}\right)}\right) + (1 - \delta_i)\left(\frac{1 - \Phi\left(\frac{y_i - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{Y_L - \mu}{\sigma}\right)}\right)\right].\end{aligned}$$

where $\delta_i = 0$ if the observation is right censored and 1 if it is not.