STA 5364, Report 1.29

Carson Slater *Baylor University*

Demonstrating 1.10.2

We have that for left-truncated observations, the likelihood function for uncensored individuals ($\delta_i = 1$) is

$$L(\cdot|\mathbf{x}) \propto \prod_{i \in \mathcal{T}_L} \frac{f(x_i)}{\Pr(X_i > Y_L)} = \prod_{i \in \mathcal{T}_{Li}} \frac{f(x_i)}{1 - F(Y_{L_i})},$$

where we only observe the time when $x \geq Y_R$. Additionally, for left truncated and right censored observations ($\delta_i = 0$), we have it that the event time X_i is greater than C_{Ri}, the censoring time. Therefore, the contribution to the likelihood is the conditional probability of surviving beyond C_{Ri} given $X_i > x_0$, which is

$$L(\cdot|\mathbf{x}) \propto \prod_{i \in \mathcal{T}_L} \frac{S(C_{Ri})}{1 - F(Y_{Li})} = \prod_{i \in \mathcal{T}_L} \frac{1 - F(y_i)}{1 - F(Y_{Li})},$$

where y_i is the observed right censored time. Additionally, for non-truncated, but right censored data (so $x_i \in \bar{\mathcal{T}}_L$ and $\delta_i = 0$), we have that

$$L(\cdot|\mathbf{x}) \propto S(C_{R_i}) = \prod_{i \in \bar{\mathcal{T}}_L} 1 - F(y_i).$$

Finally, we have that for non-truncated and non-censored data (so $x_i \in \bar{\mathcal{T}}_L$ and $\delta_i = 1$), we have that the likelihood function is

$$L(\cdot|\mathbf{x}) \propto \prod_{i \in \bar{\mathcal{T}}_L} f(x_i).$$

Putting it all together, we have it that

$$L(\cdot|\mathbf{x}) \propto \prod_{i \in \bar{\mathcal{T}}_L} [f(x_i)]^{\delta_i} [1 - F(y_i)]^{1-\delta_i} \prod_{i \in \mathcal{T}_L} \left[\frac{f(x_i)}{1 - F(Y_{L_i})}\right]^{\delta_i} \left[\frac{1 - F(y_i)}{1 - F(Y_{Li})}\right]^{1-\delta_i},$$

where $\delta_i = 0$ if the observation is right censored and 1 if it is not.