STA 5364, Report 1.27

Carson Slater *Baylor University*

2.19 (p. 61)

Consider X and Y be two competing risks with joint survival function

$$S(x, y) = (1-x)(1-y)(1+0.5xy),\ 0 < x < 1,\ 0 < y < 1.$$

(a)

To find the marginal survival function for X, we set $y = 0$, per equation 1.6.2 in the notes.

$$\begin{aligned} S_X(x) &= S(x,y)\big|_{y=0} \\ &= 1 - x, \quad 0 < x < 1. \end{aligned}$$

(b)

From equation (1.6.23) in the notes, we have that the cumulative incidence function can be written as,

$$F_j(t) = \int_0^t h_j(u) \exp\{H_T(u)\}\, du,$$

where $H_T(u) = \sum_{j=1}^K H_j(t)$ is the cumulative hazard rate for T. From this we have that

$$\begin{aligned} H_T(t) &= \int_0^t (h_1(u) + h_2(u))\, du \\ &= \int_0^t \frac{2u^2 - u + 2}{(1-u)(0.5u^2+1)}\, du \\ &= \int_0^t \left(\frac{2}{1-u} - \frac{u}{0.5u^2+1} \right) du \\ &= \left\{ -\ln\left[(1-u)^2(0.5u^2+1)\right] \right\} \Bigg|_0^t \\ &= -\ln\left[(1-t)^2(0.5t^2+1)\right]. \end{aligned}$$

So by virtue of (1.6.23),

$$\begin{aligned} F_j(t) &= \int_0^t h_j(u) \exp\{H_T(u)\}\ du \\ &= \int_0^t \left(\frac{2u^2 - u + 2}{(1-u)(0.5u^2+1)} \right) (1-u)^2(0.5u^2+1)\, du \\ &= \int_0^t (u^2 - 0.5u + 1)(1-u)\, du \\ &= \int_0^t (-u^3 + 1.5u^2 - 1.5u + 1)\, du \\ &= t - 0.75t^2 + 0.5t^3 - 0.25t^4. \end{aligned}$$