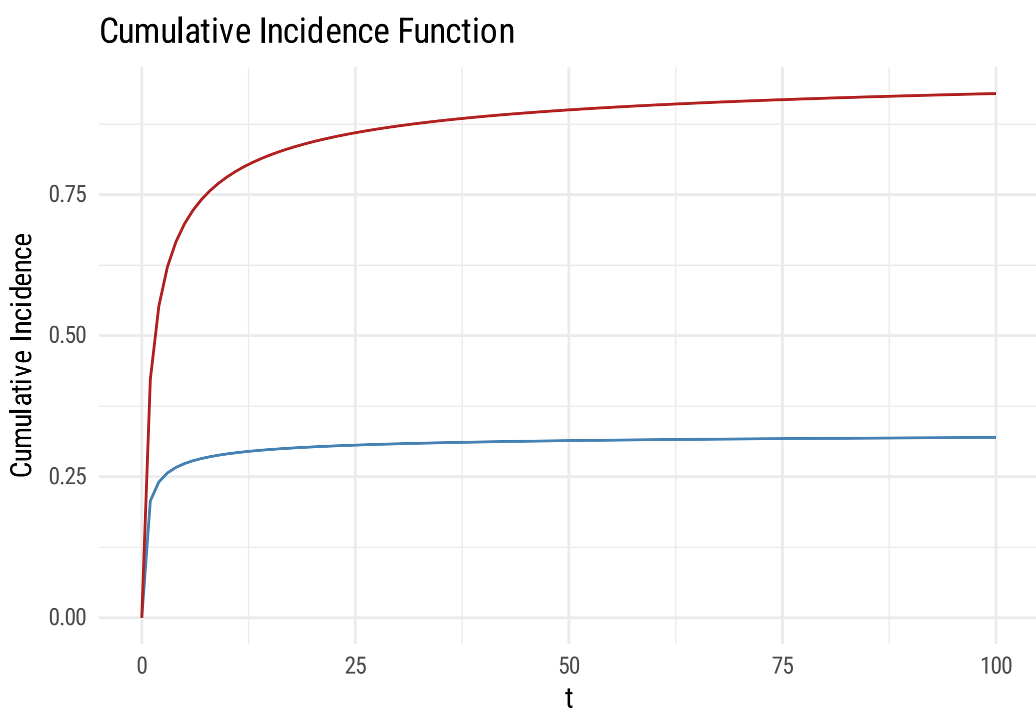
STA 5364, Report 1.26

Carson Slater *Baylor University*

Example 2.7 (p. 54)

Suppose we have three independent competing risks with joint survival function $S(t_1, t_2) = [1 + \theta(\lambda_1 t_1 + \lambda_2 t_2)]^{-1/\theta}$, $\theta \geq 0$, $\lambda_1, \lambda_2 \geq 0$. Here the crude hazard rates are given by $\lambda_i / [1 + \theta t(\lambda_1 + \lambda_2)]$, for $i = 1, 2$. The cause, specific cumulative incidence function for the ith risk is

$$
\begin{aligned}
F_i(t) &= \int_0^t \frac{\lambda_i}{1 + \theta x(\lambda_1 + \lambda_2)} \exp\left\{ - \int_0^x \frac{\lambda_1 + \lambda_2}{1 + \theta u(\lambda_1 + \lambda_2)} du \right\} dx \\
&= \int_0^t \frac{\lambda_i}{1 + \theta x(\lambda_1 + \lambda_2)} \exp\left\{ - \log[1 + \theta x(\lambda_1 + \lambda_2)]/\theta \right\} dx \\
&= \int_0^t \frac{\lambda_i}{1 + \theta x(\lambda_1 + \lambda_2)} \left(\frac{1}{[1 + \theta x(\lambda_1 + \lambda_2)]^{1/\theta}} \right) dx \\
&= \lambda_i \int_0^t \frac{1}{[1 + \theta x(\lambda_1 + \lambda_2)]^{1+1/\theta}} dx \\
&= \lambda_i \left[\frac{1}{\theta(\lambda_1 + \lambda_2)} \cdot \frac{-1}{(1 + \theta x(\lambda_1 + \lambda_2))^{1/\theta}} \right]_0^t \\
&= \frac{\lambda_i}{\theta(\lambda_1 + \lambda_2)} \left[1 - \frac{1}{[1 + \theta t(\lambda_1 + \lambda_2)]^{1/\theta}} \right] \\
&= \frac{\lambda_i}{\lambda_1 + \lambda_2} \left[1 - \frac{1}{[1 + \theta t(\lambda_1 + \lambda_2)]^{1/\theta}} \right] \\
&= \frac{\lambda_i}{\lambda_1 + \lambda_2} \left\{ 1 - [1 + \theta t(\lambda_1 + \lambda_2)]^{-1/\theta} \right\}.
\end{aligned}
$$



(Blue) Cumulative incidence function. (Red) Net probability for the first competing risk.