STA 5364, Report 1.25

Carson Slater *Baylor University*

Example 2.8 (p. 54)

Suppose we have three independent competing risks with λ_1, λ_2, λ_3, respectively. In this case, the net and crude hazard rates for the first competing risk are equal to λ_1. The hazard rate of T is $h_T(t) = \lambda_1 + \lambda_2 + \lambda_3$. The crude sub-distribution function for the first competing risk is

$$
\begin{aligned}
F_1(t) &= \int_0^t \lambda_1 \exp\left(-u(\lambda_1 + \lambda_2 + \lambda_3)\right) \, du \\
&= \lambda_1 \int_0^t \exp\left(-u(\lambda_1 + \lambda_2 + \lambda_3)\right) \, du \\
&= \lambda_1 \left[\frac{-1}{\lambda_1 + \lambda_2 + \lambda_3} \exp\left(-u(\lambda_1 + \lambda_2 + \lambda_3)\right) \right]_0^t \\
&= \lambda_1 \left(\frac{-1}{\lambda_1 + \lambda_2 + \lambda_3} \exp\left(-t(\lambda_1 + \lambda_2 + \lambda_3)\right) + \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} \exp(0) \right) \\
&= \lambda_1 \left(\frac{1}{\lambda_1 + \lambda_2 + \lambda_3} - \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} \exp\left(-t(\lambda_1 + \lambda_2 + \lambda_3)\right) \right) \\
&= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \left(1 - \exp\left(-t(\lambda_1 + \lambda_2 + \lambda_3)\right)\right).
\end{aligned}
$$

So the crude probability of death from cause 1 in the interval $[0, t]$ is not the same as the net (marginal) probability of death in this interval given by $1 - \exp\{-\lambda_1 t\}$. Also, $F_i(\infty) = \lambda_1/(\lambda_1 + \lambda_2 + \lambda_3)$, which is the probability that the first compting risk occurs first. If we consider a hypothetical world where only the first two competing risks are operating ($\mathbf{J} = \{1, 2\}$), the partial crude hazard rates are $\lambda_i'(t) = \lambda_i$, $i = 1, 2$, and the partial crude sub-division function is given by

$$
\begin{aligned}
F_1(t) &= \int_0^t \lambda_1 \exp\left(-u(\lambda_1 + \lambda_2)\right) \, du \\
&= \frac{\lambda_1}{\lambda_1 + \lambda_2} \left(1 - \exp\left(-t(\lambda_1 + \lambda_2)\right)\right).
\end{aligned}
$$

This is derived in a such a way analogous to the first derivation.