STA 5364, Report 1.22

Carson Slater *Baylor University*

2.11 (p. 59-60)

We are given the survival function of the three-parameter Weibull distribution, which is

$$S(x) = \begin{cases} 1 & \text{if } x < \phi \\ \exp\{-\lambda(x-\phi)^{\alpha}\} & \text{if } x \geq \phi. \end{cases}$$

(a)

Find the hazard rate and the density function of the three- parameter Weibull distribution.

To find the hazard rate, we employ the formula for the hazard rate

$$h(x) = -\frac{d}{dx} \ln S(x).$$

This yields the following:

$$-\frac{d}{dx} (\ln S(x)) = \lambda\alpha(x-\phi)^{\alpha-1}.$$

$$h(x) = \begin{cases} 0 & \text{if } x < \phi \\ \lambda\alpha(x-\phi)^{\alpha-1} & \text{if } x \geq \phi. \end{cases}$$

Then, using the following equality,

$$f(x) = h(x)S(x)$$

we would have that the density function would be

$$f(x) = \begin{cases} 0 & \text{if } x < \phi \\ \lambda\alpha(x-\phi)^{\alpha-1}\exp\{-\lambda(x-\phi)^{\alpha}\} & \text{if } x \geq \phi. \end{cases}$$

(b)

Suppose that the survival time X follows a three-parameter Weibull distribution with $\alpha = 1$, $\lambda = 0.0075$ and $\phi = 100$. Find the mean and median lifetimes.

The mean lifetime is given by:

$$\mathbb{E}[X] = \phi + \frac{1}{\lambda} \implies \mu = 100 + \frac{1}{0.0075} = 100 + 133.33 = 233.33.$$

The median lifetime is found by solving for t_{median} such that $S(t_{\text{median}}) = 0.5$.

$$0.5 = \exp\left(-\lambda(t_{\text{median}} - \phi)\right) \implies t_{\text{median}} = \phi - \frac{\ln(0.5)}{\lambda}$$

So then the median lifetime would be

$$t_{\text{median}} = 100 + \frac{0.6931}{0.0075} = 100 + 92.41 = 192.41.$$