STA 5364, Report 1.21

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2.9 (p. 59)

The time to relapse (in months) for patients on two treatments for lung cancer is compared using the following log-normal regression model:

$$Y = \ln(X) = 2 + 0.5Z + 2W,$$

where $W \sim \mathcal{N}(0,1)$ and $Z = 1$ if treatment A and 0 if treatment B.

(a)

Compare the survival probabilities of the two treatments at 1, 2 and 5 years.

Consider the log-normal accelerated time-to-failure (ATTF) model:

$$Y = \ln(X) = 2 + 0.5Z + 2W,$$

where: $W \sim \mathcal{N}(0,1)$ is a standard normal variable, and $Z = 1$ for treatment A and $Z = 0$ for treatment B. Since $Y = \ln(X)$, Y follows a normal distribution. We compute the distributions of Y for treatment A and treatment B as follows:

- **For treatment A** ($Z = 1$):

$$Y = 2 + 0.5(1) + 2W = 2.5 + 2W \quad \Rightarrow \quad Y \sim \mathcal{N}(2.5, 4),$$

- **For treatment B** ($Z = 0$):

$$Y = 2 + 0.5(0) + 2W = 2 + 2W \quad \Rightarrow \quad Y \sim \mathcal{N}(2, 4).$$

The survival probability for $X \leq x$ is equivalent to the probability for $Y \leq \ln(x)$, so the survival probability $S(x) = P(X > x)$ is given by:

$$S(x) = P(Y > \ln(x)).$$

Since Y is normally distributed, we can express the survival probability using the CDF of the normal distribution. For treatment A and B, the survival probabilities will differ due to the shift in the distribution of Y. The survival probability is calculated as:

$$S(x) = 1 - \Phi\left(\frac{\ln(x) - \mu_Y}{\sigma_Y}\right),$$

where μ_Y and σ_Y^2 are the mean and variance of Y for each treatment.

1. **For** $X = 1$:

$$S(1) = 1 - \Phi\left(\frac{\ln(12) - \mu_Y}{2}\right) = 1 - \Phi\left(\frac{0 - \mu_Y}{2}\right).$$

2. **For** $X = 2$:

$$S(2) = 1 - \Phi\left(\frac{\ln(2) - \mu_Y}{2}\right) = 1 - \Phi\left(\frac{\ln(24) - \mu_Y}{2}\right).$$

3. **For** $X = 5$:

$$S(5) = 1 - \Phi\left(\frac{\ln(5) - \mu_Y}{2}\right) = 1 - \Phi\left(\frac{\ln(60) - \mu_Y}{2}\right).$$

You can substitute the respective values of μ_Y for treatment A ($\mu_Y = 2.5$) and treatment B ($\mu_Y = 2$) to calculate the survival probabilities for each case. We do that in the table below of survival probabilities.

	1 Year	2 Years	5 Years
Treatment A	0.5030	0.3673	0.2127
Treatment B	0.4042	0.2779	0.1475

(b)

Repeat the calculations if W has a standard logistic distribution.

If W has a standard logistic distribution, Y follows a logistic distribution. For treatment A, we have $Y \sim \mathcal{L}(2.5, 2)$, where $2W$ follows a logistic distribution with location 2.5 and scale parameter 2. For treatment B, we have therefore, $Y \sim \mathcal{L}(2, 2)$, where $2W$ follows a logistic distribution with location 2 and scale parameter 2.

The survival probability $S(x) = P(X > x) = 1 - P(Y \leq \ln(x))$ is given by the complementary CDF of the logistic distribution:

$$S(x) = \frac{1}{1 + e^{\frac{\ln(x) - \mu_Y}{s}}},$$

where μ_Y is the location (mean) and s is the scale of the logistic distribution. Here, we calculate survival probabilities for different values of X:

1. **For** $X = 1$:

$$S(1) = \frac{1}{1 + e^{\frac{\ln(12) - \mu_Y}{2}}}.$$

2. **For** $X = 2$:

$$S(2) = \frac{1}{1 + e^{\frac{\ln(24) - \mu_Y}{2}}} = \frac{1}{1 + e^{\frac{0.693 - \mu_Y}{2}}}.$$

3. **For** $X = 5$:

$$S(5) = \frac{1}{1 + e^{\frac{\ln(60) - \mu_Y}{2}}} = \frac{1}{1 + e^{\frac{1.609 - \mu_Y}{2}}}.$$

You can substitute the values of μ_Y for treatment A ($\mu_Y = 2.5$) and treatment B ($\mu_Y = 2$) to calculate the survival probabilities for each case.

	1 Year	2 Years	5 Years
Treatment A	0.5019	0.4160	0.3106
Treatment B	0.4397	0.3569	0.2598

Compared with (a), when W has a logistic distribution, the survival probabilities are lower for both treatments over the years.