STA 5364, Report 1.16

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2.3 (p. 58)

The time to death (in days) following a kidney transplant follows a log-logistic distribution with $\alpha = 1.5$ and $\lambda = 0.01$.

(a)

Find the 50, 100, and 150 day survival probabilities for kidney transplantation in patients. The survival function for the log-logistic distribution is given by:

$$S(t) = \frac{1}{1 + \lambda\, t^{\alpha}} = \frac{1}{1 + 0.01\; t^{1.5}}.$$

So, for times, $t = 50, 100$, and 150, we have that the probabilities of survival are 0.22, 0.091, and 0.052 respectively.

(b)

Find the median time to death following a kidney transplant.

We have that the median survival time is the value of t which satisfies

$$0.5 = \frac{1}{1 + 0.01\; t^{1.5}},$$

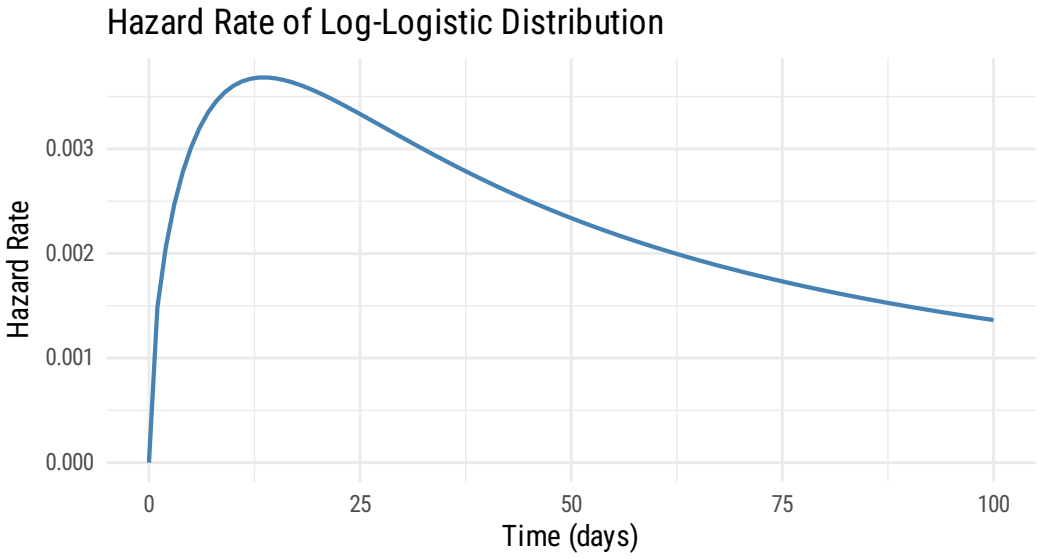
which is $t \approx 21.5$.

(c)

Show that the hazard rate is initially increasing and, then, decreasing over time. Find the time at which the hazard rate changes from increasing to decreasing.

We have that the hazard rate of this phenomena would be

$$h(t) = \frac{\lambda^{\alpha} \alpha t^{\alpha - 1}}{1 + \lambda t^{\alpha}}.$$



Clearly from the graphic, the hazard rate increases and then decreases. Using `optimize()`, we can find the maximum value of the hazard function, which yields $t = 13.57$ days as the point where the hazard rate begins to decrease.

(d)

Find the mean time to death.

The mean time to death for a log-logistic distribution, according to page 38 in the text, is

$$\mu = \frac{\pi \operatorname{Csc}(\pi/\alpha)}{\alpha \lambda^{1/\alpha}} \quad \text{if } \alpha > 1.$$

Which yields $\mu = 52.10283$ days when $\alpha = 1.5$ and $\lambda = 0.01$.