STA 6384, Report 4.7

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Reproducing Example 4.4.2: Analysis of Political Ideology, Gender, and Political Party Affiliation

1. Introduction and Background

This report reproduces the analysis presented in Example 4.4.2 from the course notes on categorical data analysis. The example examines data from the 1991 General Social Survey, which cross-classifies respondents based on three categorical variables:

- **Gender (G)**: Female (F) or Male (M).
- **Political Party (P)**: Democrat (D) or Republican (R).
- **Political Ideology (I)**:
  - Very Liberal (VL).
  - Slightly Liberal (SL).
  - Moderate (M).
  - Slightly Conservative (SC).
  - Very Conservative (VC).

The primary objective is to fit loglinear (Poisson) models to investigate the associations among these variables, assess model fit, and estimate odds ratios for key interactions. All analyses were conducted using R, following the code provided in Section 8.18.3 of the notes.

2. Data Description

The dataset consists of counts of respondents in each combination of gender, political party, and ideology. The first few rows of the dataset are shown below for illustration:

Gender	Party	Ideology	Count
F	D	VL	44
F	D	SL	47
F	D	M	118
F	D	SC	23

Gender	Party	Ideology	Count
F	D	VC	32
F	R	VL	18

The full dataset is derived from Figure 4.5 in the notes and includes all combinations of the variables.

3. Models Fitted

The following loglinear models were fitted to the data:

1. **Saturated Model (GPI)**:
   Includes the full three-way interaction:
$$\log \mu_{ijk} = \lambda + \lambda_i^G + \lambda_j^P + \lambda_k^I + \lambda_{ij}^{GP} + \lambda_{ik}^{GI} + \lambda_{jk}^{PI} + \lambda_{ijk}^{GPI}.$$
2. **Homogeneous Association Model (GP, GI, PI)**:
   Includes all two-way interactions but no three-way interaction:
$$\log \mu_{ijk} = \lambda + \lambda_i^G + \lambda_j^P + \lambda_k^I + \lambda_{ij}^{GP} + \lambda_{ik}^{GI} + \lambda_{jk}^{PI}.$$
3. **Conditional Independence Model (G, PI)**:
   Assumes gender is independent of the other variables given party and ideology:
$$\log \mu_{ijk} = \lambda + \lambda_i^G + \lambda_j^P + \lambda_k^I + \lambda_{jk}^{PI}.$$
4. **Mutual Independence Model (G, P, I)**:
   Assumes all variables are independent:
$$\log \mu_{ijk} = \lambda + \lambda_i^G + \lambda_j^P + \lambda_k^I.$$

Models were fitted using the `glm()` function in R with a Poisson family.

4. Model Comparisons Using Likelihood Ratio Tests

Models were compared using deviance statistics and likelihood ratio tests (via `anova()`). The results are summarized below:

Comparison	df	Deviance	p-value	Conclusion
(GP, GI, PI) vs. (GPI)	4	3.245	0.518	Fail to reject null: No three-way interaction needed.
(G, PI) vs. (GP, GI, PI)	5	14.273	0.014	Reject null: GP and GI interactions contribute to the fit.

Comparison	df	Deviance	p-value	Conclusion
(G, P, I) vs. (G, PI)	4	62.333	< 2.2e-16	Reject null: PI interaction is important.

Based on these results, the homogeneous association model (GP, GI, PI) was selected as the working model, as it provides an adequate fit without the need for a three-way interaction.

5. Hypothesis Tests for Terms in the Homogeneous Association Model

Type II likelihood ratio tests were performed using `Anova()` from the `car` package to assess the significance of each term in the homogeneous association model:

Term	LR Chisq	df	p-value	Significance Note
gender	20.637	1	5.55e-06	Highly significant (***)
party	0.528	1	0.467	Not significant
ideol	154.131	4	< 2.2e-16	Highly significant (***)
gender:party	3.530	1	0.060	Marginally significant
gender:ideol	8.965	4	0.062	Marginally significant
party:ideol	60.555	4	2.22e-12	Highly significant (***)

These tests indicate strong evidence for the party-ideology (PI) interaction, with marginal evidence for gender-party (GP) and gender-ideology (GI) interactions.

6. Odds Ratio Estimation Using Profile Likelihood Confidence Intervals

Odds ratios were estimated for the two-way interactions under the homogeneous association model. Profile likelihood confidence intervals were computed using `mcprofile()`, with single-step adjustment for multiplicity within each family of contrasts (GP: 1 interval; GI and PI: 10 intervals each).

GP Interaction: Odds of Republican Affiliation (Males vs. Females)

Contrast	OR	95% CI (Adjusted)
GP Rep|M:F	1.32	0.988 – 1.76

GI Interaction: Ideology Odds Ratios (Males vs. Females)

Contrast	OR	95% CI (Adjusted)
GI VC:SC|M:F	0.92	0.475 – 1.77
GI VC:M|M:F	1.57	0.906 – 2.71

Contrast	OR	95% CI (Adjusted)
GI VC:SL|M:F	1.24	0.634 – 2.42
GI VC:VL|M:F	1.08	0.536 – 2.18
GI SC:M|M:F	1.71	0.949 – 3.08
GI SC:SL|M:F	1.35	0.668 – 2.73
GI SC:VL|M:F	1.18	0.566 – 2.46
GI M:SL|M:F	0.79	0.440 – 1.43
GI M:VL|M:F	0.69	0.372 – 1.29
GI SL:VL|M:F	0.87	0.424 – 1.80

PI Interaction: Ideology Odds Ratios (Republicans vs. Democrats)

Contrast	OR	95% CI (Adjusted)
PI VC:SC|R:D	0.88	0.439 – 1.76
PI VC:M|R:D	2.02	1.17 – 3.54
PI VC:SL|R:D	3.12	1.60 – 6.25
PI VC:VL|R:D	4.78	2.31 – 10.25
PI SC:M|R:D	2.28	1.26 – 4.24
PI SC:SL|R:D	3.54	1.74 – 7.40
PI SC:VL|R:D	5.40	2.52 – 12.08
PI M:SL|R:D	1.55	0.864 – 2.82
PI M:VL|R:D	2.37	1.24 – 4.68
PI SL:VL|R:D	1.53	0.712 – 3.35

Observation: Most PI odds ratios exceed 1 and increase with greater ideological distance, indicating Republicans are more likely to identify with conservative ideologies relative to Democrats, as expected.

7. Wald Confidence Intervals

Wald confidence intervals (without multiplicity adjustment) were also computed and are generally similar to the profile likelihood intervals. For example, for PI VC:VL|R:D:
- Point Estimate: 4.78
- Wald 95% CI: (2.80, 8.15)
- Profile Likelihood 95% CI (Adjusted): (2.31, 10.25)

8. Model Parameter Estimates

The following table summarizes the parameter estimates from the homogeneous association model, presented as log incidence rate ratios (log(IRR)) with 95% confidence intervals and p-values:

Characteristic	log(IRR)	95% CI	p-value
gender: F	—	—	—
gender: M	-0.70	-0.97, -0.44	<0.001
party: D	—	—	—
party: R	-0.24	-0.49, 0.00	0.050
ideol: M	—	—	—
ideol: SC	-1.6	-2.0, -1.3	<0.001
ideol: SL	-0.83	-1.1, -0.53	<0.001
ideol: VC	-1.3	-1.7, -0.97	<0.001
ideol: VL	-0.90	-1.2, -0.59	<0.001
gender * party: M * R	0.28	-0.01, 0.56	0.060
gender * ideol: M * SC	0.53	0.11, 0.96	0.013
gender * ideol: M * SL	0.24	-0.19, 0.66	0.3
gender * ideol: M * VC	0.45	0.05, 0.84	0.026
gender * ideol: M * VL	0.37	-0.08, 0.82	0.10
party * ideol: R * SC	0.83	0.40, 1.3	<0.001
party * ideol: R * SL	-0.44	-0.87, -0.02	0.044
party * ideol: R * VC	0.70	0.31, 1.1	<0.001
party * ideol: R * VL	-0.86	-1.3, -0.39	<0.001

Abbreviations: CI = Confidence Interval; IRR = Incidence Rate Ratio.

9. Conclusions

- There is no evidence of a three-way interaction (GPI), supporting the use of the homogeneous association model (GP, GI, PI) as an adequate representation of the data.
- The main effects of gender and ideology are highly significant, while the party main effect is not.
- Interactions:
  - Strong evidence exists for the party × ideology (PI) interaction.
  - Marginal evidence supports the gender × ideology (GI) and gender × party (GP) interactions.
- Odds ratio interpretations:
  - Males have slightly higher odds of Republican affiliation compared to females, though the confidence interval includes 1.
  - Male vs. female ideology odds ratios are generally centered near 1, with some exceeding 1 for conservative categories.
  - Republican vs. Democrat ideology odds ratios exceed 1 for most conservative vs. liberal comparisons, aligning with expected political alignments.

This reproduction aligns closely with the results in Example 4.4.2, with minor numerical differences attributable to rounding or software defaults.
