STA 6384, Report 1.7

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Problem: Work problem 1.9, p. 30 of Agresti.

In an experiment on chlorophyll inheritance in maize, for 1103 seedlings of self-fertilized heterozygous green plants, 854 seedlings were green and 249 were yellow. Theory predicts the ratio of green to yellow is 3:1. Test the hypothesis that 3:1 is the true ratio. Report the p-value, and interpret.

We will test the hypothesis that the observed data fits the theoretical 3:1 ratio using a Chi-squared (χ^2) goodness-of-fit test.

The null hypothesis (H_0) is that the observed ratio of green to yellow seedlings is consistent with the theoretical ratio of 3:1. The alternative hypothesis (H_a) is that the ratio is not 3:1.

- H_0: The true ratio of green to yellow seedlings is 3:1.
- H_a: The true ratio of green to yellow seedlings is not 3:1.

The total number of seedlings is $N = 854 + 249 = 1103$. Based on the 3:1 ratio, we expect the seedlings to be $\frac{3}{4}$ green and $\frac{1}{4}$ yellow.

$$\text{Expected Green} = \frac{3}{4} \times 1103 = 827.25$$
$$\text{Expected Yellow} = \frac{1}{4} \times 1103 = 275.75$$

The Chi-squared test statistic is calculated using the formula:

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

We can organize the calculations in a table:

Category	**Green**	**Yellow**
Observed (O)	854	249
Expected (E)	827.25	275.75
(O – E)	26.75	-26.75
$(O - E)^2$	715.5625	715.5625
$\frac{(O-E)^2}{E}$	$\frac{715.5625}{827.25} \approx 0.8650$	$\frac{715.5625}{275.75} \approx 2.5950$

Now, we sum the last row to find the χ^2 statistic:

$$\chi^2 = 0.8650 + 2.5950 = 3.46$$

The degrees of freedom (df) for a goodness-of-fit test are calculated as $df = (\text{number of categories} - 1)$. In this case, $df = 2 - 1 = 1$.

We look for the probability of obtaining a χ^2 value of 3.46 or greater with 1 degree of freedom.

$$p\text{-value} = P(\chi^2_{df=1} \geq 3.46)$$

Using a χ^2 distribution calculator or table, we find:

$$p\text{-value} \approx 0.0629$$

We compare the p-value to a chosen significance level, $\alpha = 0.05$.

Since the p-value (0.0629) is greater than the significance level ($\alpha = 0.05$), we fail to reject the null hypothesis. While the p-value is not small enough to reject the null at the 0.05 level, it is marginal (around 0.06).

All in all, there is not sufficient statistical evidence to reject the hypothesis that the true ratio of green to yellow seedlings is 3:1. The observed results are consistent with the theoretical model of chlorophyll inheritance.