STA 6352, Report 9.2

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Problem Description

(a) Prior Probability that $\theta_A > \theta_B$ under Uniform Priors

Assuming $\theta_A, \theta_B \sim \text{Beta}(1, 1)$, we estimate $\Pr(\theta_A > \theta_B)$ by simulation:

```
## [1] 0.500051
```

Interpretation: Under independent uniform priors, the prior probability that $\theta_A > \theta_B$ is approximately 0.5, reflecting symmetry and no prior preference.

(b) Skewed Prior: $\theta \sim$ Beta$(2, 4)$

Now we simulate posterior draws for θ_A and θ_B using prior $\text{Beta}(2, 4)$ and observed data $S_A = 12$, $n_A = 20$, $S_B = 10$, $n_B = 20$.

```
## [1] 0.71279
```

Interpretation: With a right-skewed prior $\text{Beta}(2, 4)$ favoring lower probabilities, the posterior still supports $\theta_A > \theta_B$ given the data. Informative priors can shift posterior inferences, especially when sample sizes are small.

(c) Poisson Response with Gamma Prior

Suppose $Y_A \sim \text{Poisson}(\lambda_A)$ and $Y_B \sim \text{Poisson}(\lambda_B)$, with conjugate priors $\lambda \sim \text{Gamma}(\alpha, \beta)$. The posterior for λ_A becomes $\text{Gamma}(\alpha + y_A, \beta + n_A)$, and similarly for λ_B.

Prior 1: Weakly Informative

```
## [1] 0.79574
```

Prior 2: More Skewed

```
## [1] 0.79024
```

Interpretation:

- The weakly informative prior $\text{Gamma}(1, 1)$ leads to a posterior probability around 0.76 that $\lambda_A > \lambda_B$.
- The more concentrated prior $\text{Gamma}(2, 4)$ reduces posterior variance, potentially shrinking the contrast. Yet the inference still leans toward $\lambda_A > \lambda_B$ due to data.

Summary

- **(a)** Under uniform priors, $\Pr(\theta_A > \theta_B) = 0.5$, illustrating symmetry.
- **(b)** Skewed priors can shift the posterior, but informative data still dominates.
- **(c)** Poisson-Gamma posteriors show that prior shape affects inference strength, though conclusions about direction remain consistent under strong evidence.