

STA 6352, Report 7.9

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Exercise 14.11 in BTiB

The data and program statements to analyze the data from the toenail study (Example 14.5) are on the book's website for this chapter. The files have `toenail` in their names.

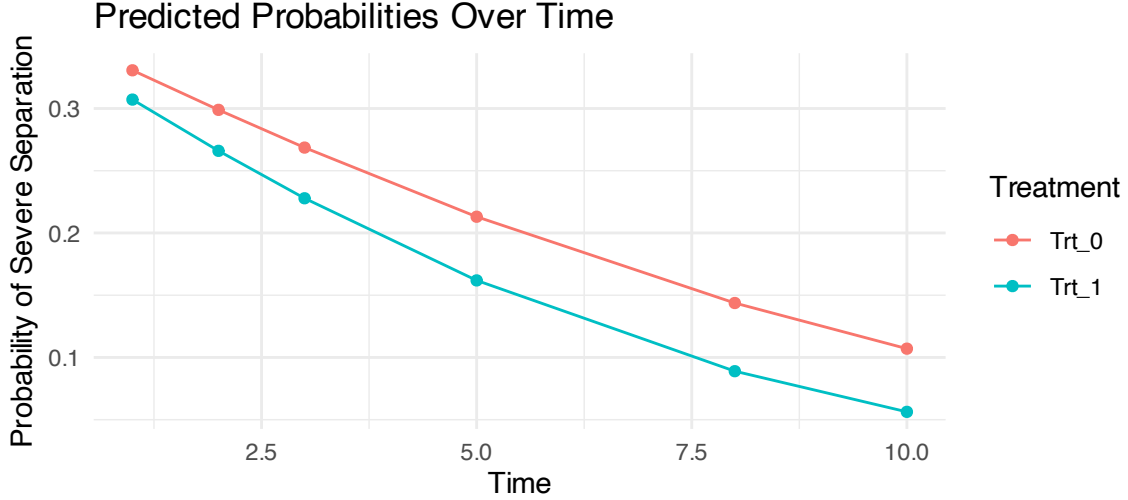
- (a) Modify the code so that you can create a table of inferences that provides predicted probabilities of moderate or severe separation at times 1, 2, 3, 5, 8, and 10 for a new individual who receives $Trt = 1$ with $\gamma_{38+1} | \sigma_g^2 \sim \mathcal{N}(3.1, \sigma_g^2)$. Repeat the process for the same individual, assuming now that this person receives $Trt = 0$. The table should be 6×2 . Compare the probabilities across times and treatments.
- (b) Now perform your own sensitivity analysis and comment.

Part (a)

Here is the table requested.

Table 1: Predicted Probabilities for New Individual

Time	Trt_0	Trt_1
1	0.331	0.307
2	0.299	0.266
3	0.269	0.228
5	0.213	0.162
8	0.144	0.089
10	0.107	0.056



Sensitivity Analysis

This sensitivity analysis examines how posterior predictive probabilities change under different prior assumptions for the variance parameter τ in a hierarchical Bayesian logistic regression model. Both models predict the probability of a binary outcome $y_i \sim \text{Bernoulli}(p_i)$, where

$$\text{logit}(p_i) = \gamma_{\text{ID}[i]} + \beta_1 \cdot \text{Trt}_i + \beta_2 \cdot \text{Time}_i + \beta_3 \cdot \text{Trt}_i \cdot \text{Time}_i.$$

Here, $\gamma_j \sim \text{Normal}(\mu, \tau^{-1})$ captures subject-level random effects, and the regression coefficients $\beta_1, \beta_2, \beta_3$ have diffuse priors $\text{Normal}(0, 100)$. The difference between the two models lies in the prior on the precision τ : *Model A* uses a moderately informative prior $\tau \sim \text{Gamma}(2, 0.5)$, while *Model B* adopts a much weaker prior $\tau \sim \text{Gamma}(0.01, 0.01)$, which allows for a wider range of possible values for the subject-level variability.

Posterior samples from each model are used to simulate predictive probabilities across a grid of time points and treatment arms. The resulting estimates are plotted to visually assess how sensitive the model-based predictions are to the prior choice on τ . Differences between the two models' predicted probabilities are modest, suggesting that the model's posterior inferences are relatively robust to the prior specification on the hierarchical variance.

Table 2: Posterior Predictive Probabilities Across Models

Time	Model_A_Tr0	Model_A_Tr1	Model_B_Tr0	Model_B_Tr1
1	0.316	0.292	0.318	0.292
2	0.285	0.251	0.286	0.251
3	0.255	0.213	0.256	0.213
5	0.200	0.149	0.200	0.148
8	0.133	0.080	0.133	0.080
10	0.099	0.050	0.099	0.051

