STA 6352, Report 7.5

Carson Slater *Baylor University*

Problem

Reproduce the results of Example 14.1 in BTIB. In addition to the output provided in the text, provide posterior density plots and convergence diagnostics.

Hierarchical Model

The observed conception rates for bull i in series j, denoted as Y_{ij}, are assumed to follow a normal distribution:

$$Y_{ij} \mid \mu_i, \sigma^2 \sim \mathcal{N}(\mu_i, \sigma^2),$$

where μ_i represents the latent mean conception rate for bull i and σ captures the within-bull variability.

To account for between-bull variability, the latent means μ_i are modeled hierarchically:

$$\mu_i \mid \mu, \sigma_\alpha^2 \sim \mathcal{N}(\mu, \sigma_\alpha^2),$$

where μ is the overall mean conception rate, and σ_α represents the between-bull standard deviation.

Bayesian Inference

We specify diffuse priors:

$$\mu \sim \text{Uniform}(0, 100), \quad \sigma \sim \text{Uniform}(0, 100), \quad \sigma_\alpha \sim \text{Uniform}(0, 100).$$

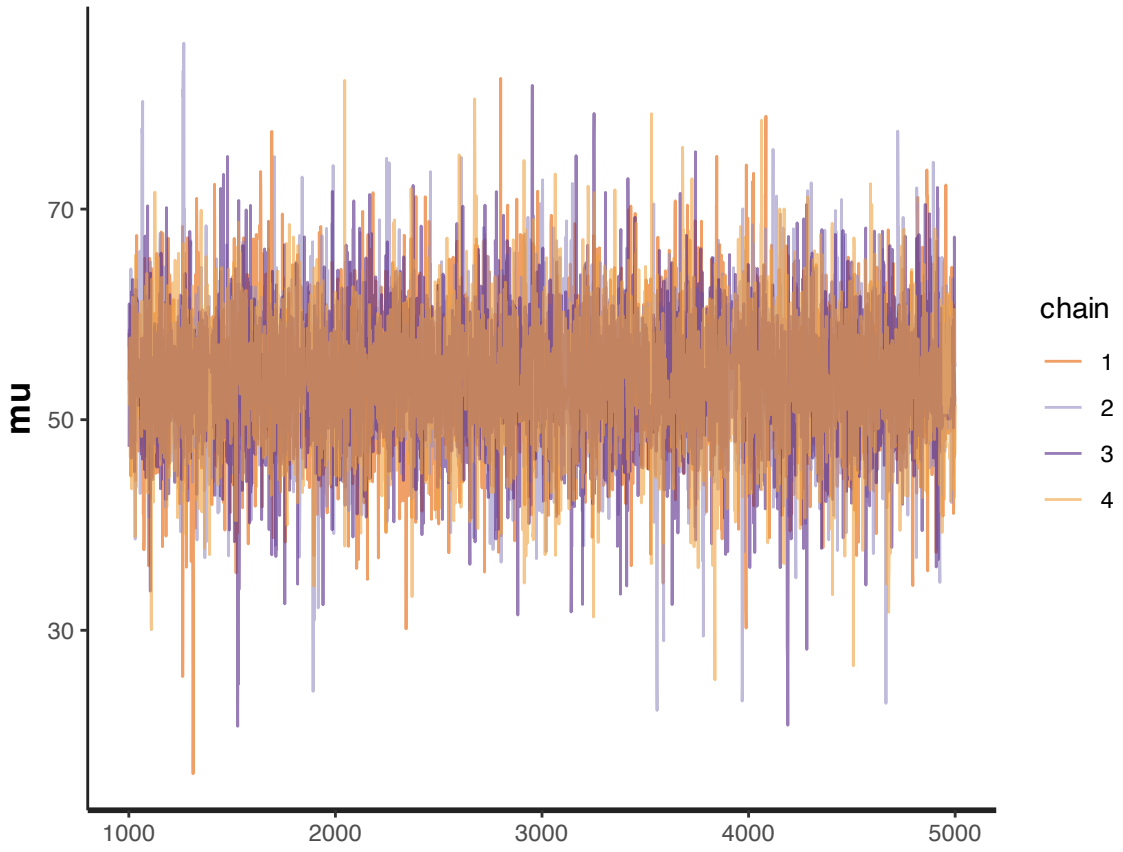
Markov Chain Monte Carlo (MCMC) sampling via JAGS is used to estimate the posterior distributions of $\mu, \sigma, \sigma_\alpha$, and the latent means μ_i.

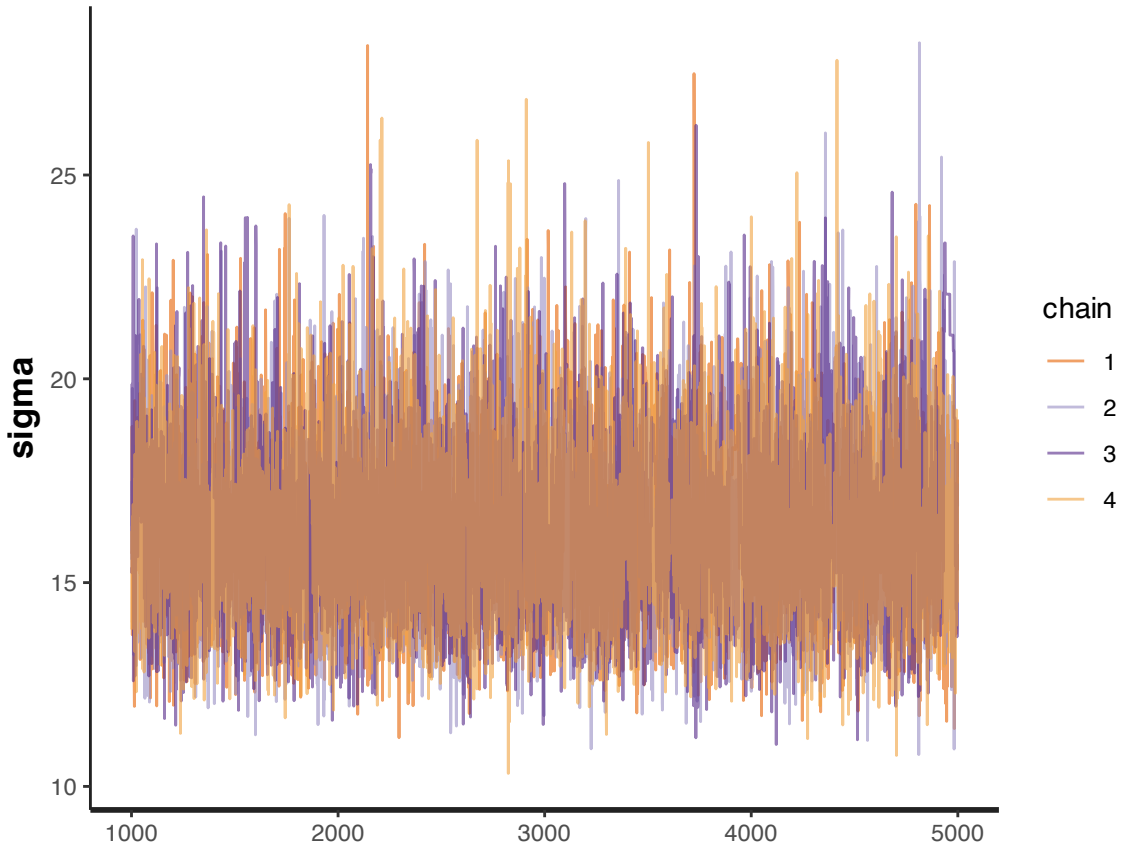
Results and Diagnostics

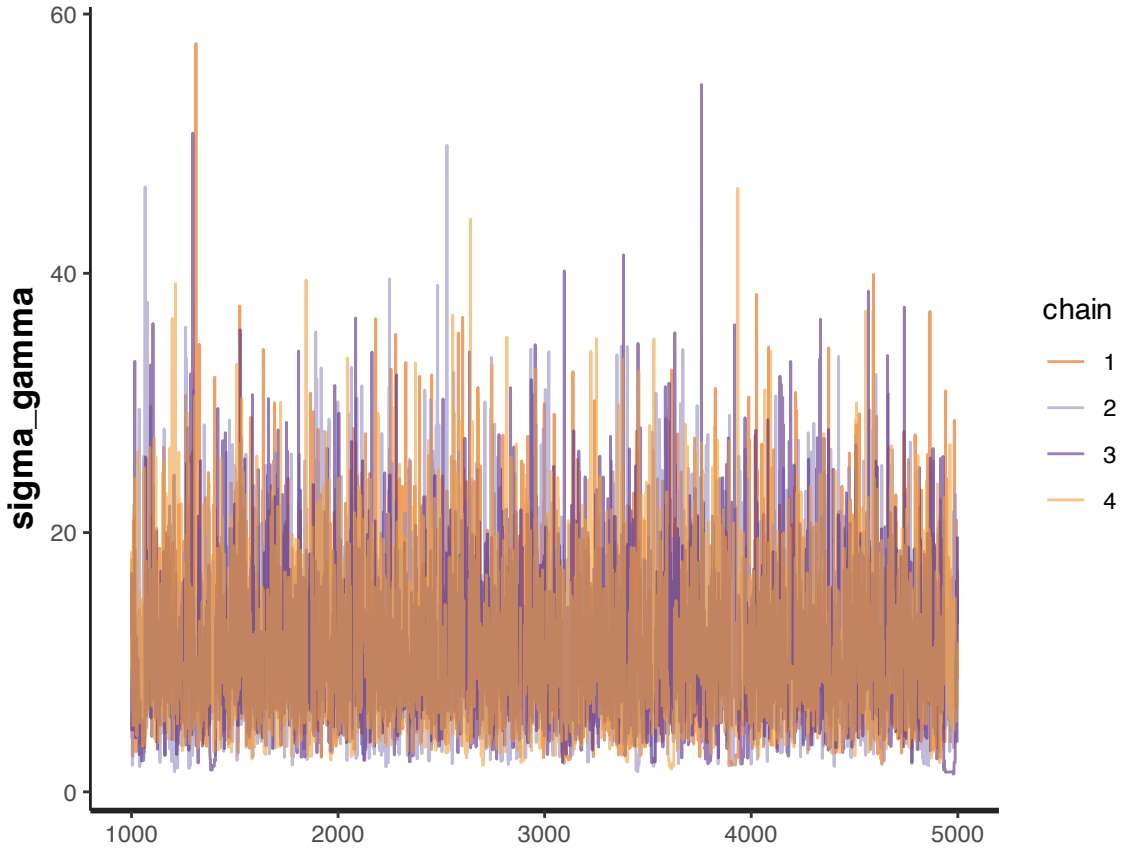
The posterior median and 95% credible intervals for the key parameters are:

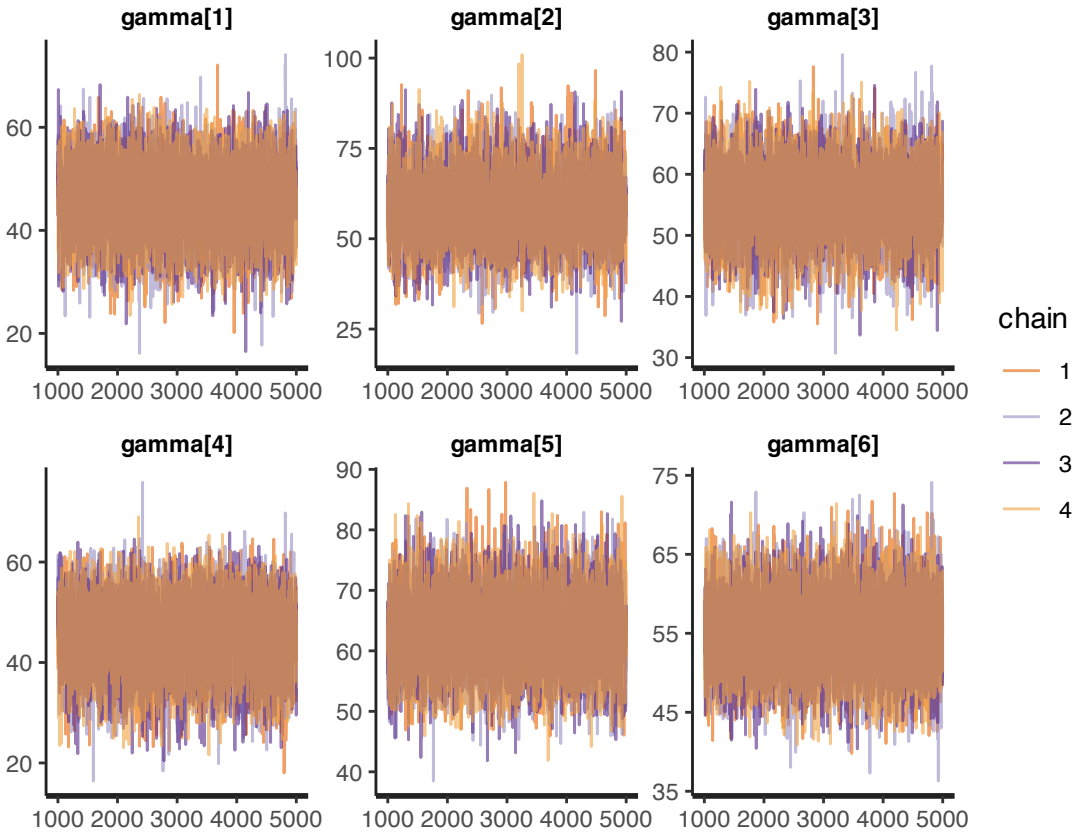
- Overall mean conception rate: $\mu = 53.82$ (41.93, 65.94)
- Within-bull standard deviation: $\sigma = 16.37$ (12.82, 24.9)
- Between-bull standard deviation: $\sigma_\alpha = 10.69$ (2.63, 24.90)

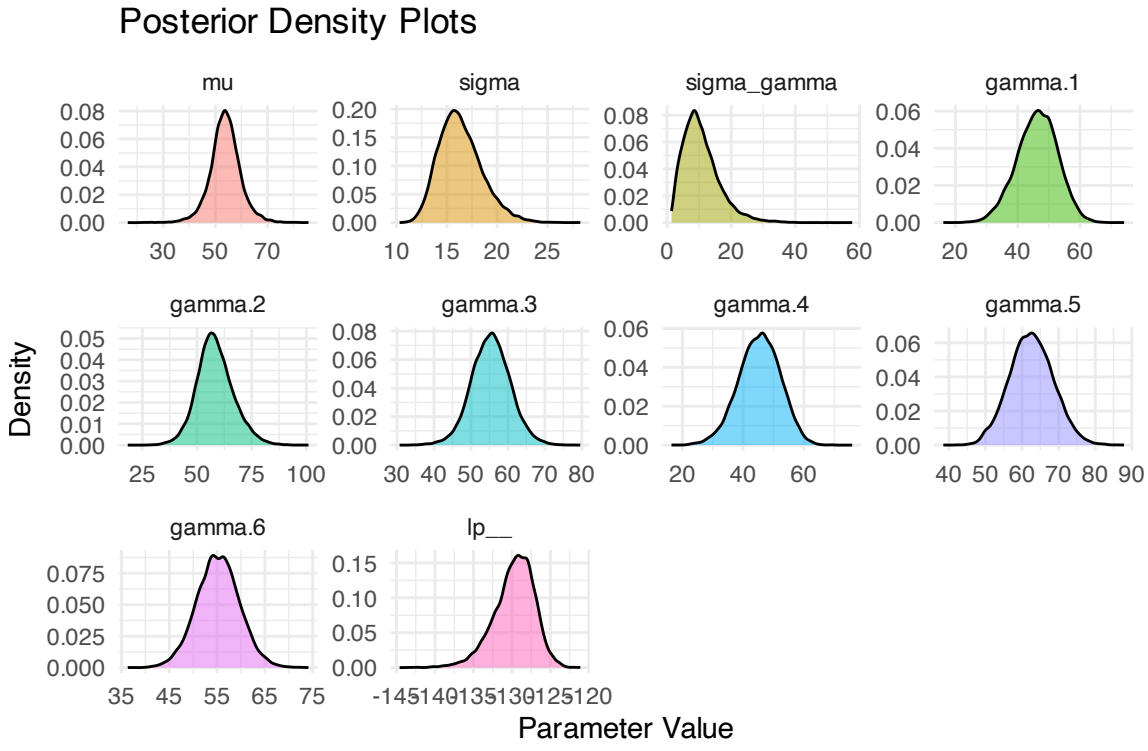
	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%
mu	53.8179	0.0682	5.8770	41.9340	50.4382	53.7937	57.1741	65.9378
sigma	16.3661	0.0302	2.1442	12.8229	14.8375	16.1403	17.6486	21.1571
sigma_-gamma	10.6942	0.0960	5.7341	2.6270	6.6148	9.6902	13.6259	24.8969
gamma[1]	46.2459	0.0837	6.5487	32.8538	41.9240	46.4574	50.7654	58.4542
gamma[2]	58.3205	0.0869	8.3670	42.7674	52.8544	57.7762	63.3433	76.2907
gamma[3]	55.3813	0.0508	5.2707	45.0069	51.8993	55.3911	58.8091	65.8518
gamma[4]	45.2794	0.0922	6.8304	31.3500	40.7632	45.5092	50.0552	58.0770
gamma[5]	62.5977	0.0900	5.9798	50.9557	58.5037	62.5037	66.6040	74.5000
gamma[6]	55.1989	0.0412	4.4633	46.3931	52.1978	55.1933	58.1416	64.0566




sigma
25
20
15
10
1000
2000
3000
4000
5000
chain
1
2
3
4


sigma_gamma
60
40
20
0
1000
2000
3000
4000
5000
chain
1
2
3
4


gamma[1]
gamma[2]
gamma[3]
gamma[4]
gamma[5]
gamma[6]
1000 2000 3000 4000 5000
chain
1
2
3
4


Posterior Density Plots
mu
sigma
sigma_gamma
gamma.1
gamma.2
gamma.3
gamma.4
gamma.5
gamma.6
lp__
Density
Parameter Value