STA 6352, Report 7.4

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Problem

Rework the example in section 7.2.1, except suggest a mildly informative prior on μ_θ.

Normal Hierarchical Model with Unknown Mean and Variance

We consider a hierarchical model for cholesterol levels in a population of females aged 60-70 years with type 2 diabetes and hypertension. Let y_{ij} be the cholesterol measurement (mg/dL) for subject j at clinic i, where $i = 1, \ldots, 8$ and $j = 1, \ldots, 5$.

We assume the following structure:

$$y_{ij} \mid \theta_i, \sigma_y^2 \sim \mathcal{N}(\theta_i, \sigma_y^2)$$

$$\theta_i \mid \mu_\theta, \sigma_\theta^2 \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$$

where both the mean and variance are unknown.

For the priors, we set:

$$\sigma_y \sim \mathcal{U}(0, B_1), \quad \sigma_\theta \sim \mathcal{U}(0, B_2)$$

$$\mu_\theta \sim \mathcal{N}(100, 10^2)$$

where we use a mildly informative prior for μ_ω based on expected cholesterol levels.

Stan Implementation

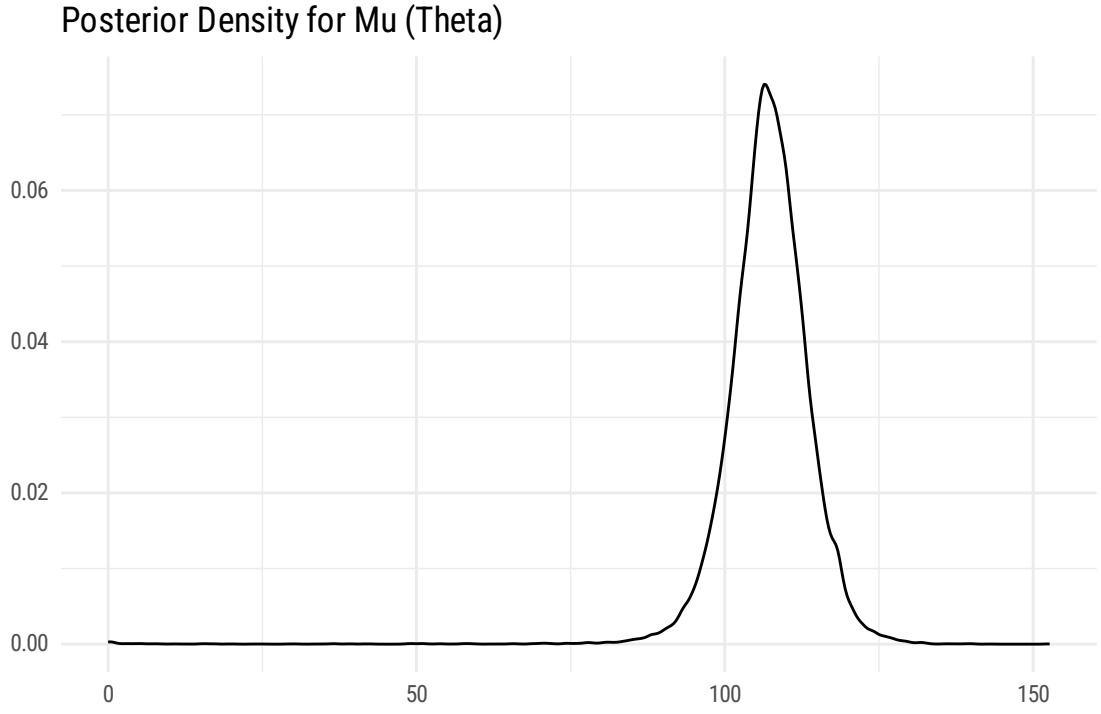
Table 1: Posterior results for the simple normal hierarchical model.

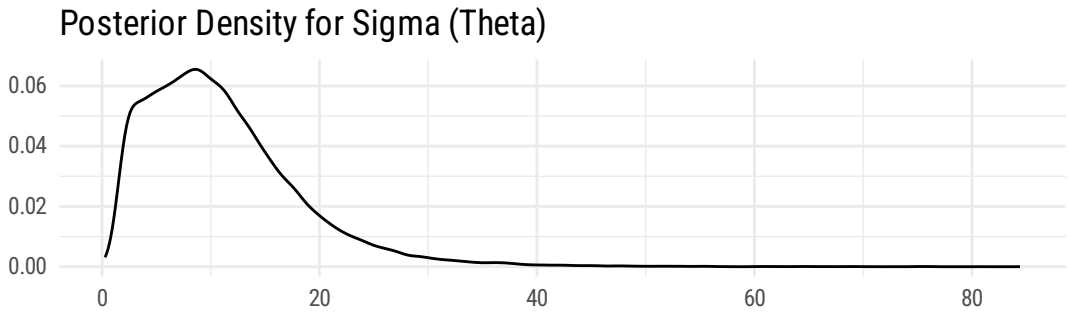
	Mean	SD	0.025 Quantile	0.975 Quantile
Theta 1	110.7482	9.313574	95.15847	128.3033
Theta 2	115.0996	10.26281	99.1239	135.2426
Theta 3	110.991	9.333969	95.52587	128.7013

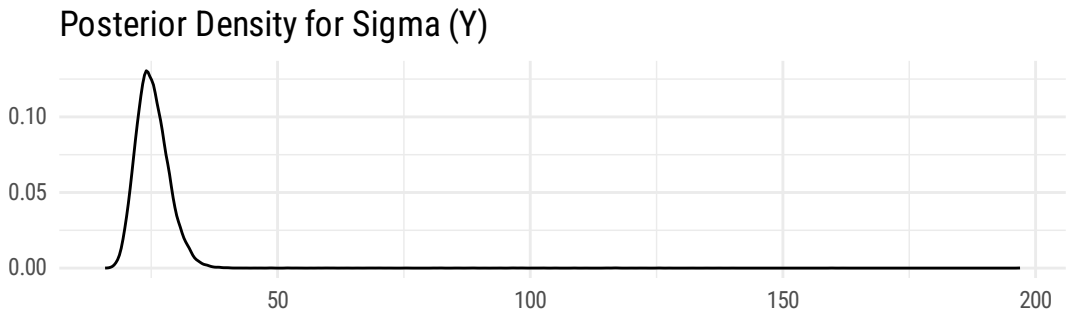
	Mean	SD	0.025 Quantile	0.975 Quantile
Theta 4	99.04681	10.17091	78.40918	115.8219
Theta 5	99.27919	10.03715	78.61384	115.761
Theta 6	110.2303	9.192808	94.72249	127.4562
Theta 7	108.777	9.114894	92.71347	125.4643
Theta 8	103.802	9.218353	86.10273	119.3517
Mu (Theta)	106.9975	7.494043	94.18	119.1192
Sigma (Theta)	11.10567	7.219314	1.938877	28.68716
Sigma (Y)	25.44349	4.913644	19.88727	32.52154

Table 2: Posterior results for the simple normal independent model.

	Mean	SD	0.025 Quantile	0.975 Quantile
Theta 1	113.3283	13.06975	88.39137	136.1774
Theta 2	122.8762	15.26478	90.49163	146.721
Theta 3	112.9816	15.89049	85.77579	136.7005
Theta 4	86.34221	14.5046	59.95151	109.8937
Theta 5	86.15033	16.58928	55.46624	109.9819
Theta 6	111.4921	15.14464	82.49908	135.1952
Theta 7	108.4361	14.62971	79.90135	131.8288
Theta 8	96.94888	14.29847	65.06164	120.4217
Sigma (Y)	26.37307	8.039835	19.78089	38.49449




Posterior Density for Sigma (Theta)
0.06
0.04
0.02
0.00
0
20
40
60
80


Posterior Density for Sigma (Y)
0.10
0.05
0.00
50
100
150
200