STA 6352, Report 6.2

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Problem

Reproduce the results of the last example. Estimate the joint mean and $P(x < 0.5, y < 0.5)$.

Example 6.5

To sample from a bivariate distribution $f(x, y) = \frac{3}{2\pi}\sqrt{1 - x^2 - y^2}$ over $x^2 + y^2 \leq 1$, construct a MC that provides uniform observations from the subgraph $\mathcal{C} = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq f(x, y)\}$.

```r
set.seed(613)  # For reproducibility

# Number of samples
n <- 10000

# Initialize storage for samples
samples <- matrix(0, nrow = n, ncol = 3)
colnames(samples) <- c("x", "y", "z")

# Initialize starting values
samples[1, ] <- c(0, 0, runif(1, 0, 3/(2*pi)))

# Sampling loop
for (i in 2:n) {
  # Sample z
  x_prev <- samples[i - 1, 1]
  y_prev <- samples[i - 1, 2]
  f_xy_prev <- (3 / (2 * pi)) * sqrt(max(0, 1 - x_prev2 - y_prev2))
  z <- runif(1, 0, f_xy_prev)

  # Sample y
  y_range_sq <- max(0, (-9 * z2 - 4 * pi2 * x_prev2 + 9))
  y_range <- (1/3) * sqrt(y_range_sq)
  y <- runif(1, -y_range, y_range)

  # Sample x
  x_range_sq <- max(0, (-9 * z2 - 4 * pi2 * y2 + 9))
  x_range <- (1/3) * sqrt(x_range_sq)
  x <- runif(1, -x_range, x_range)
```

```r
  # Store values
  samples[i, ] <- c(x, y, z)
}

# Estimate the joint mean
joint_mean <- colMeans(samples[, 1:2], na.rm = TRUE)

# Estimate probability P(x < 0.5, y < 0.5)
prob_estimate <- mean(samples[, 1] < 0.5 & samples[, 2] < 0.5, na.rm = TRUE)

# Print results
cat("Estimated Joint Mean:", joint_mean, "\n")
```

```
## Estimated Joint Mean: 0.001422011 0.002168356
```

```r
cat("Estimated Probability P(x < 0.5, y < 0.5):", prob_estimate, "\n")
```

```
## Estimated Probability P(x < 0.5, y < 0.5): 0.7212
```