STA 6360, Report 5.2

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Derivation of Distance for Beta/Binomial Model

The distance function $\delta(m, \bar{\theta}, \phi, q_0)$ for the Beta/Binomial model is given by:

$$\delta(m, \bar{\theta}, \phi, q_0) = \left((\tilde{\alpha} - 1)\bar{\theta}^{-2} + (\tilde{\beta} - 1)(1 - \bar{\theta})^{-2}\right) - \left(\frac{\tilde{\alpha}}{c + \sum_{y=0}^{m} Y f_m(Y_m) - 1}\bar{\theta}^{-2} + \frac{\tilde{\beta}}{c + m - \sum_{y=0}^{m} Y f_m(Y_m) - 1}(1 - \bar{\theta})^{-2}\right),$$

where:

- $f_m(Y_m) = \text{BeBin}(n, \tilde{\alpha}, \tilde{\beta})$,
- $\bar{\theta} = \mathbb{E}_p(\theta) = \frac{\alpha}{\alpha+\beta}$,
- c is a normalizing constant, and
- $(\tilde{\alpha}, \tilde{\beta})$ are updated hyperparameters after observing data.

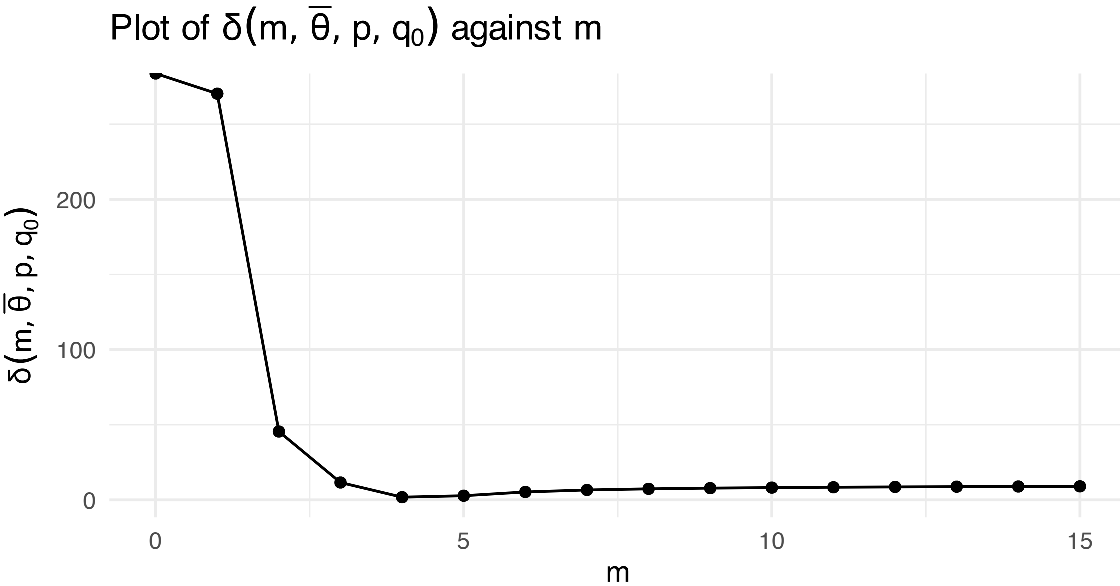
Steps of Derivation

1. **Prior Contribution:** The first term corresponds to the contribution of the prior information:
$$(\tilde{\alpha} - 1)\bar{\theta}^{-2} + (\tilde{\beta} - 1)(1 - \bar{\theta})^{-2}.$$
2. **Posterior Contribution:** The second term corresponds to the contribution of the posterior information:
$$\frac{\tilde{\alpha}}{c + \sum_{y=0}^{m} Y f_m(Y_m) - 1}\bar{\theta}^{-2} + \frac{\tilde{\beta}}{c + m - \sum_{y=0}^{m} Y f_m(Y_m) - 1}(1 - \bar{\theta})^{-2}.$$
3. **Distance:** The difference between the prior and posterior contributions gives the distance $\delta(m, \bar{\theta}, \phi, q_0)$.

Explanation of Terms

- $(\tilde{\alpha}, \tilde{\beta})$ are parameters of the Beta distribution, updated to reflect the data.
- $f_m(Y_m)$ represents the Binomial likelihood weighted by the prior.
- $\bar{\theta} = \frac{\alpha}{\alpha+\beta}$ is the prior mean of θ, reflecting its expected value under the Beta prior.

This formulation captures the balance between the information provided by the prior and the updated posterior distribution in terms of its effect on the expected distance.



I could not seem to replicate the desired results.