# STA 6360, Report 4.5

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*1*

Work Problem 12 on p. 82 in *BDA3*.

Poisson regression model: expand the model of Exercise 2.13(a) by assuming that the number of fatal accidents in year $t$ follows a Poisson distribution with mean $\alpha + \beta t$. You will estimate $\alpha$ and $\beta$ following th example of the analysis in Section 3.7.

*(a)*

Discuss various choices for a 'noninformative' prior for ($\alpha$, $\beta$). Choose one.

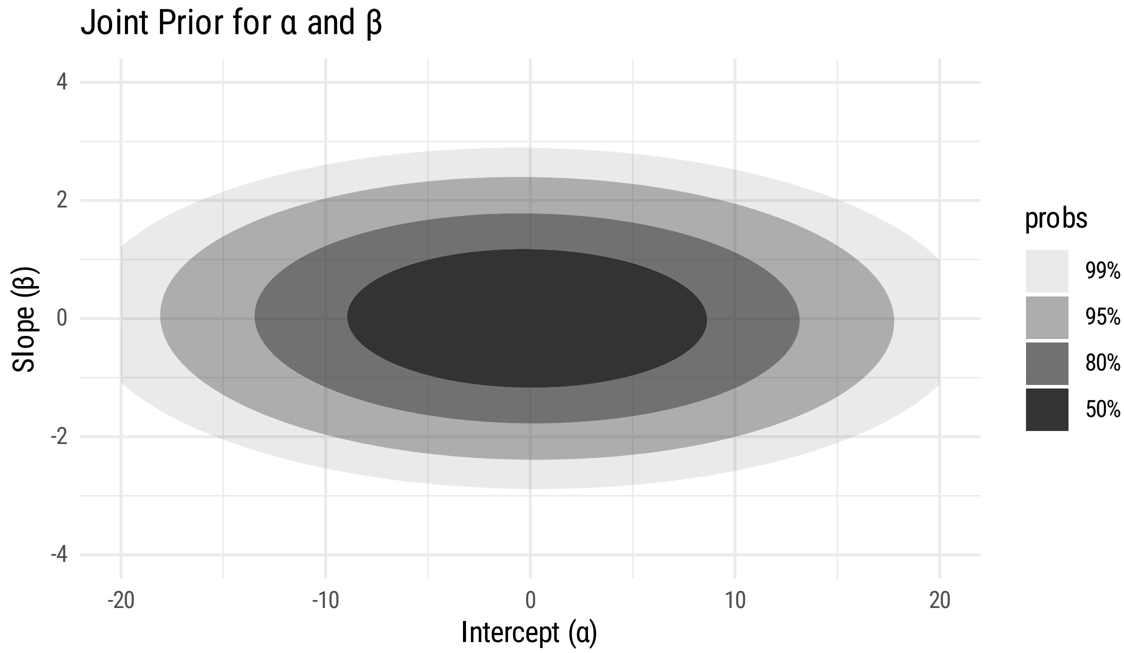
Choices for non-informative priors on $\alpha$ would include be a diffuse normal or $t$ distribution. A uniform prior on a specified range could also work too. For example for each parameter you could specify a $\mathcal{N}(0, 1000)$ prior.

a) Discuss choices for a noninformative prior for $(\alpha, \beta)$. Several choices for noninformative priors are:
    - **Uniform prior:** $p(\alpha, \beta) \propto 1$. This assumes that all values of $\alpha$ and $\beta$ are equally likely.
    - **Independent improper priors:** $p(\alpha) \propto 1$ and $p(\beta) \propto 1$. This assumes that $\alpha$ and $\beta$ are independent and have no prior information.
    - **Jeffreys' prior:** $p(\alpha, \beta) \propto \sqrt{|\det(I(\alpha, \beta))|}$, where $I(\alpha, \beta)$ is the Fisher information matrix. This prior is invariant under reparameterization.

*(b)*

Discuss what would be a realistic informative prior distribution for ($\alpha$, $\beta$). Sketch its contours and then put it aside. Do parts (c)–(h) of this problem using your noninformative prior distribution from (a).

A good informative prior would be (without using the data to ellicit such a prior) would be a normal prior of the coefficients. Let's give $\alpha \sim \mathcal{N}(0, 10)$ and $\beta \sim \mathcal{N}(0, 1)$.




*(c)*

The likelihood function for the Poisson regression model is:

$$L(\alpha, \beta \mid y, t) = \prod_{i=1}^{n} \frac{e^{-(\alpha+\beta t_i)}(\alpha + \beta t_i)^{y_i}}{y_i!}.$$

Using the uniform prior, the posterior density is proportional to the likelihood:

$$p(\alpha, \beta \mid y, t) \propto \prod_{i=1}^{n} e^{-(\alpha+\beta t_i)}(\alpha + \beta t_i)^{y_i}.$$

*(d)*

To verify that the posterior density is proper, we need to confirm that it integrates to a finite value:

$$\int \int p(\alpha, \beta \mid y, t)\, d\alpha\, d\beta < \infty.$$

Since the likelihood function is a product of Poisson densities, which are always non-negative and integrate to 1, the posterior density is proper.

*(e)*

Below are the estimates and the data.

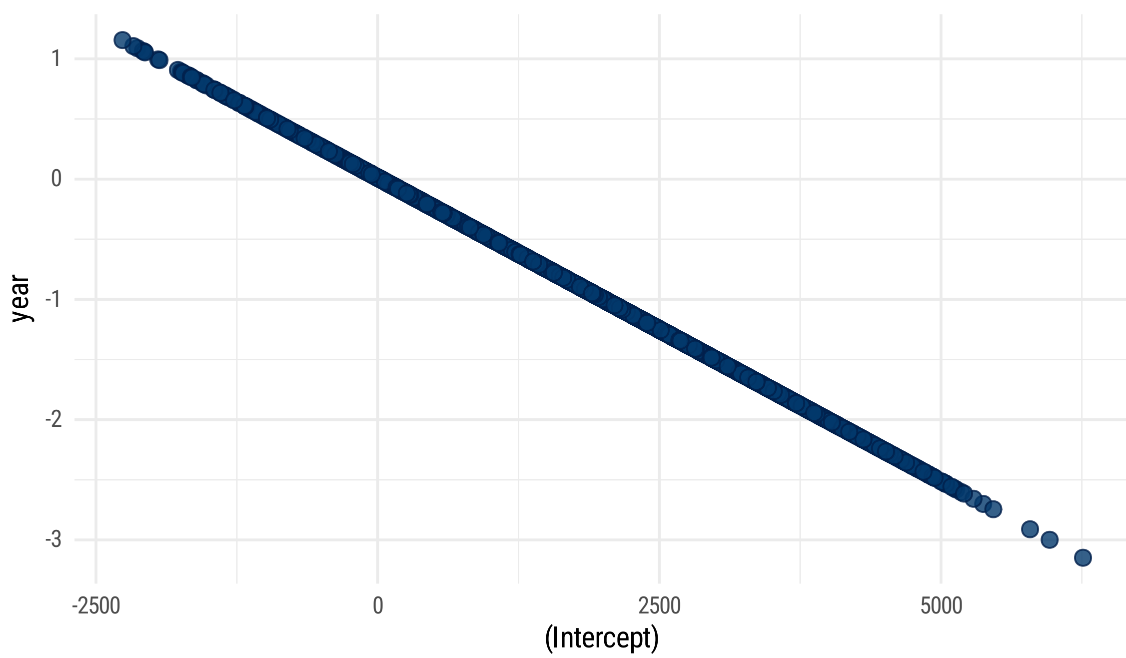
Table 1: Data for airline accidents.

| Year | Fatal Accidents | Passenger Deaths | Death Rate |
|---|---|---|---|
| 1976 | 24 | 734 | 0.19 |

| Year | Fatal Accidents | Passenger Deaths | Death Rate |
|---|---|---|---|
| 1977 | 25 | 516 | 0.12 |
| 1978 | 31 | 754 | 0.15 |
| 1979 | 31 | 877 | 0.16 |
| 1980 | 22 | 814 | 0.14 |
| 1981 | 21 | 362 | 0.06 |
| 1982 | 26 | 764 | 0.13 |
| 1983 | 20 | 809 | 0.13 |
| 1984 | 16 | 223 | 0.03 |
| 1985 | 22 | 1066 | 0.15 |

| | mean | mcse | sd | 10% | 50% | 90% | n_eff | Rhat |
|---|---|---|---|---|---|---|---|---|
| (Intercept) | 1906.8917109 | 23.6272654 | 1088.5175449 | 503.112523 | 1914.2483105 | 3312.67945 | 2122 | 1.0008621 |
| year | -0.9507192 | 0.0119361 | 0.5495214 | -1.660642 | -0.9545589 | -0.24175 | 2120 | 1.0008638 |
| mean_PPD | 23.9948429 | 0.0263876 | 2.2096866 | 21.200000 | 23.9000000 | 26.80000 | 7012 | 0.9999823 |
| log-posterior | -34.4617194 | 0.0213731 | 1.0524175 | -35.768241 | -34.1446416 | -33.52503 | 2425 | 1.0005033 |

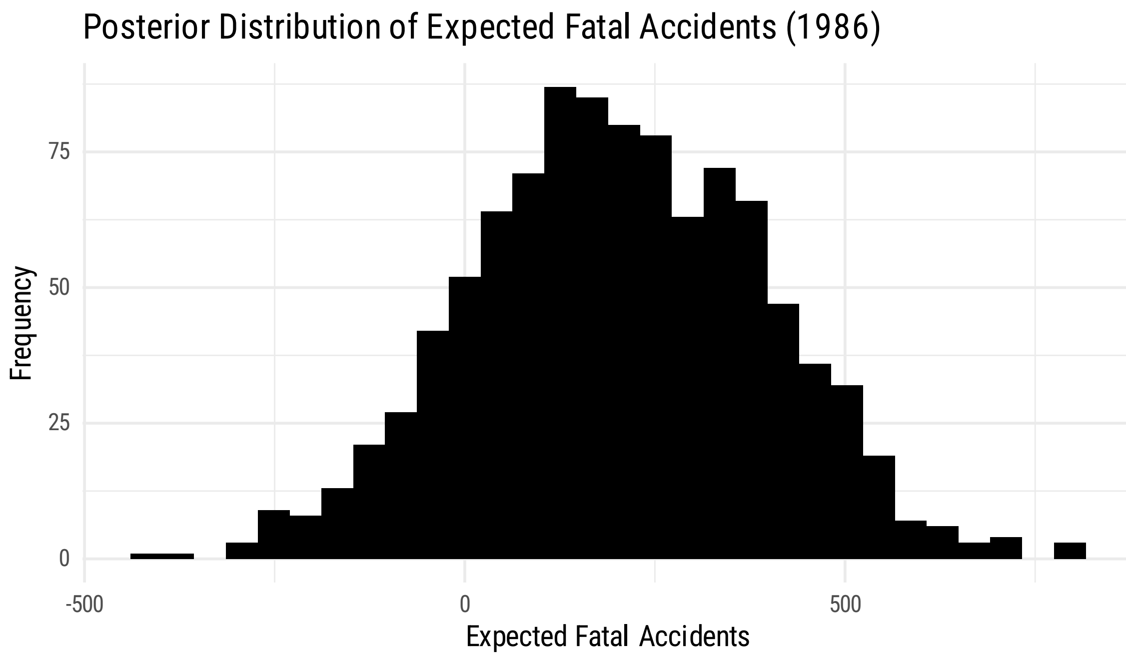
*(f)*




Could not figure out how to plot the posterior density using this so I used a scatter plot. It did not go well.

*(g)*

The left plot are intercept draws and right are coefficient draws.




*(h)*

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## [1] "95% Predictive Interval: 21.1"
## [2] "95% Predictive Interval: 574.8"
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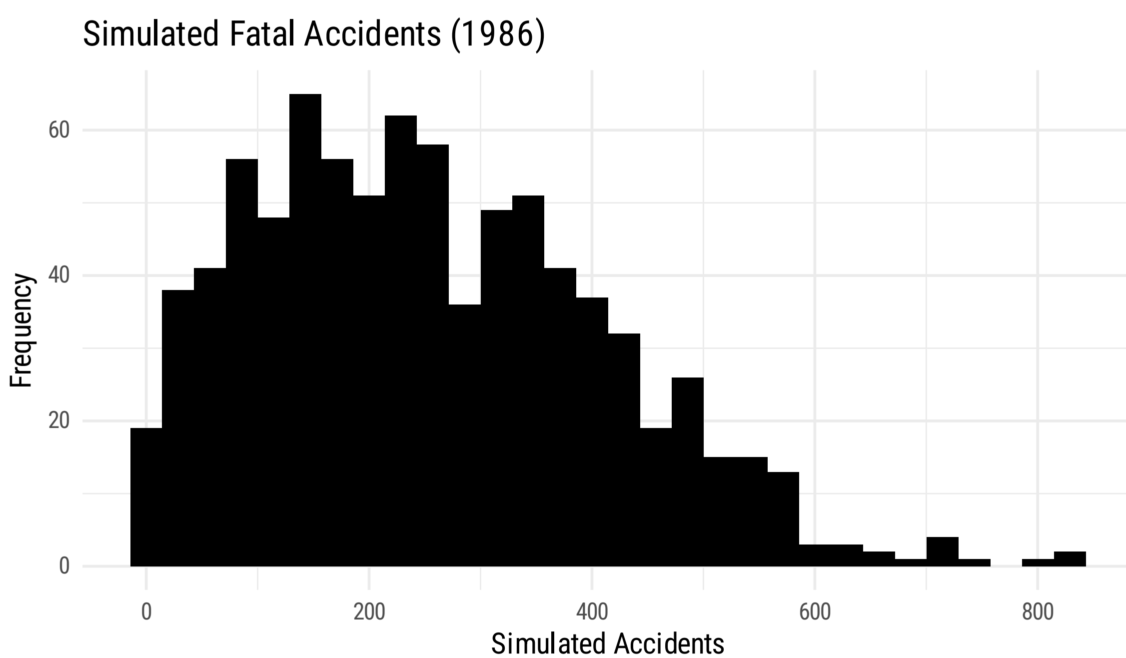
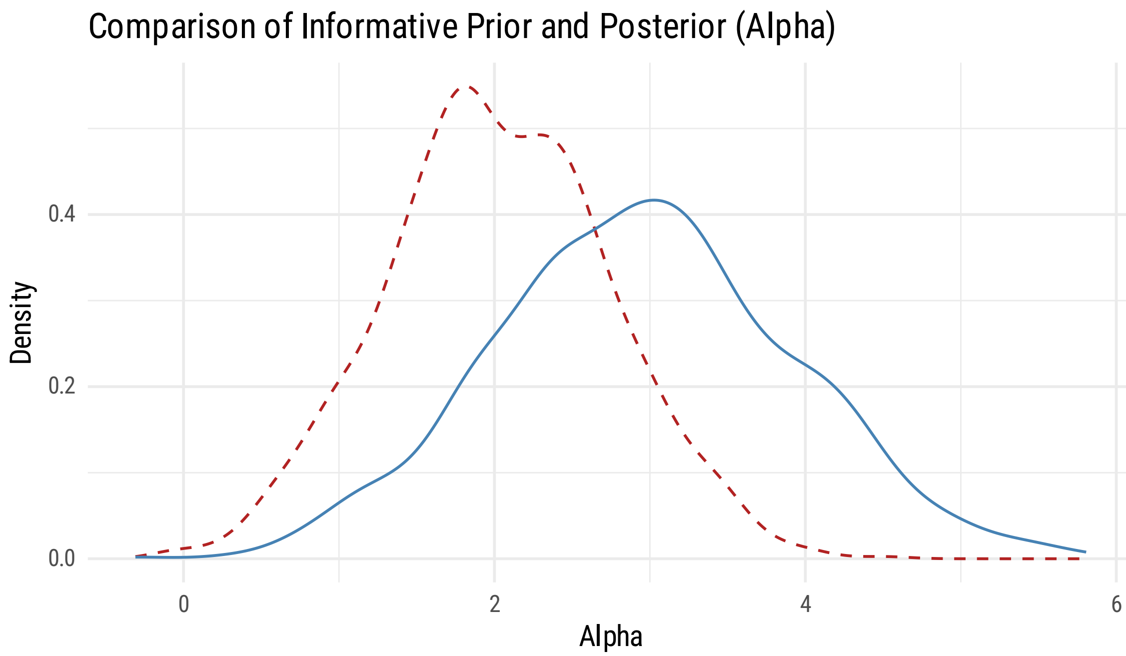
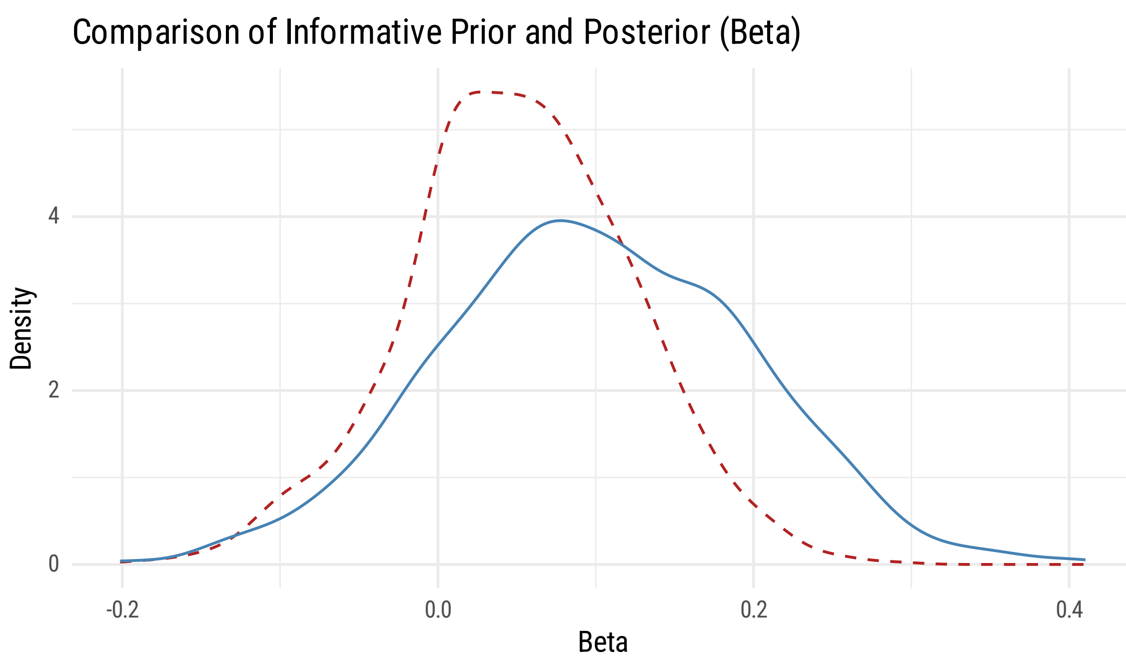


*(i)*


Comparison of Informative Prior and Posterior (Alpha)
Density
0.4
0.2
0.0
0
2
4
6
Alpha


Comparison of Informative Prior and Posterior (Beta)
Density
4
2
0
-0.2
0.0
0.2
0.4
Beta