

STA 6360, Report 4.1

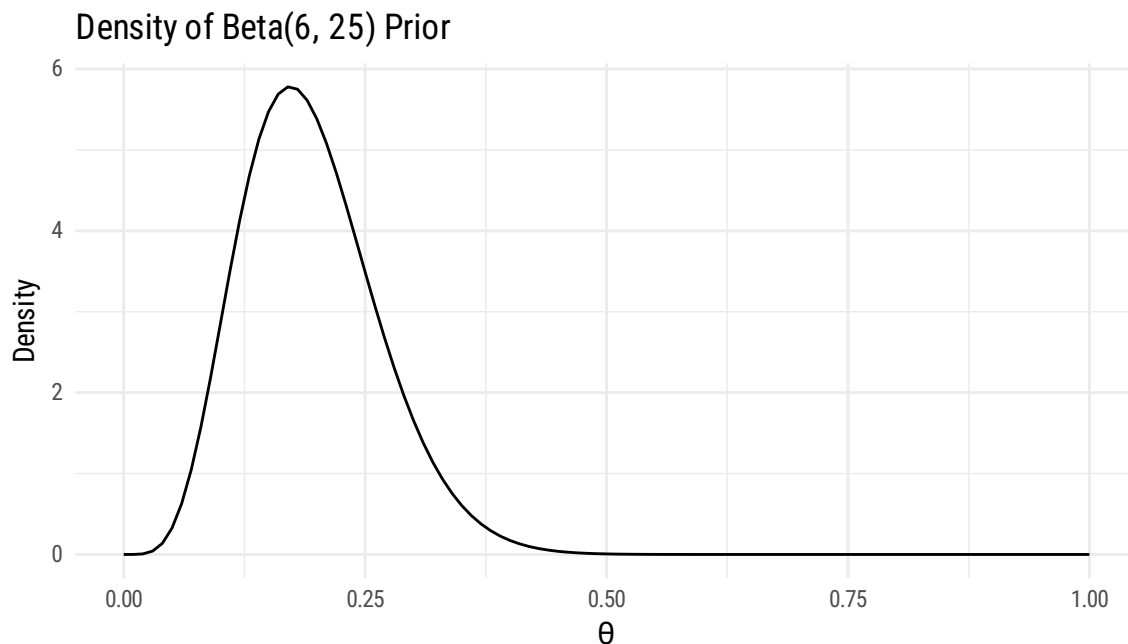
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Problem

Suppose 30 subjects in a study each record the number of days (out of 20) that they experience at least one migraine headache. You decide to use a binomial distribution to model this data, with probability θ . A clinician with expertise in this area is consulted and provides that, for the population in question, the chance is most likely about 20% that a subject experiences a migraine on any one day—and this number might be as large as 30%.

1.

Select a Beta prior to reflect the experts opinion and justify your choice.



Using the guidelines on page 228 in the notes, I chose a prior by imposing $\mu = 0.20$ with a standard deviation of $\sigma = 0.05$. I felt this left room for values beyond the 20% and 30% elicited values, but focused the prior in the center of that. I used the following formulas to elicit shape values:

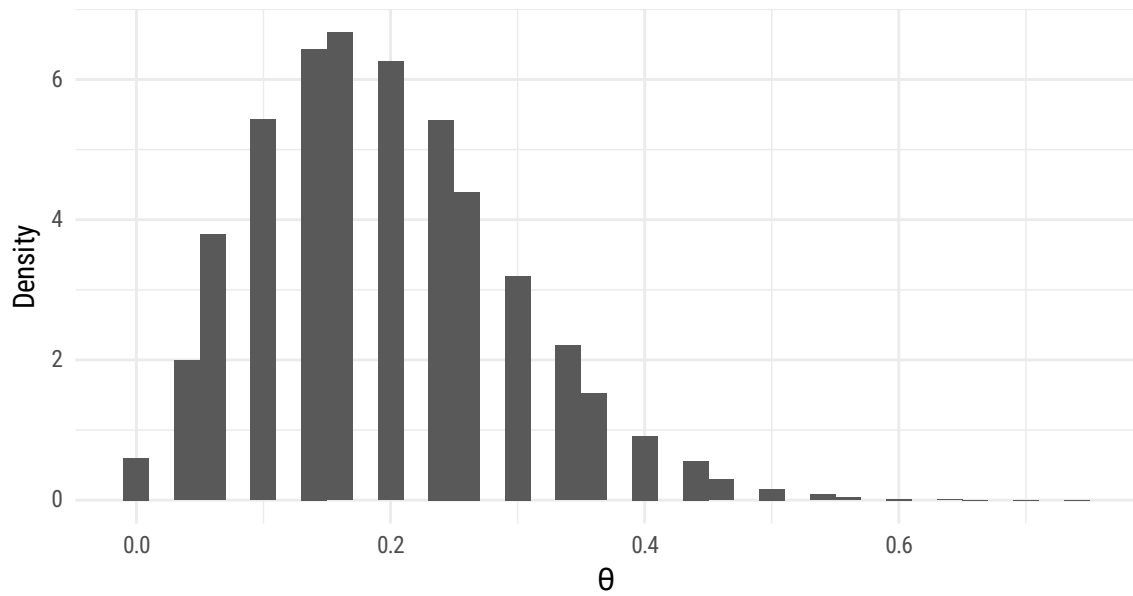
$$a = \frac{-\mu(\sigma^2 + \mu^2 - \mu)}{\sigma^2} \quad \text{and} \quad b = \frac{\mu - 1(\sigma^2 + \mu^2 - \mu)}{\sigma^2}.$$

These yielded the shape parameters $a = 12.6$ and $b = 50.4$. This particular prior would be analogous to having observed approximately $n = 63$ observations in another study, which feels too specific. We then divided the original shape parameters by almost 2, and chose the shape parameters $a = 6$ and $b = 25$, which seemed to yield a similar prior.

2.

Use the prior predictive distribution to generate a sample that can be used as feedback to the expert. Provide the feedback graphically.

100,000 Simulated Samples and Their Probabilities



3.

What is the ESS of the prior? Does this seem reasonable?

The ESS of the prior is $n = 31$. To be completely honest, this does not seem reasonable.

4.

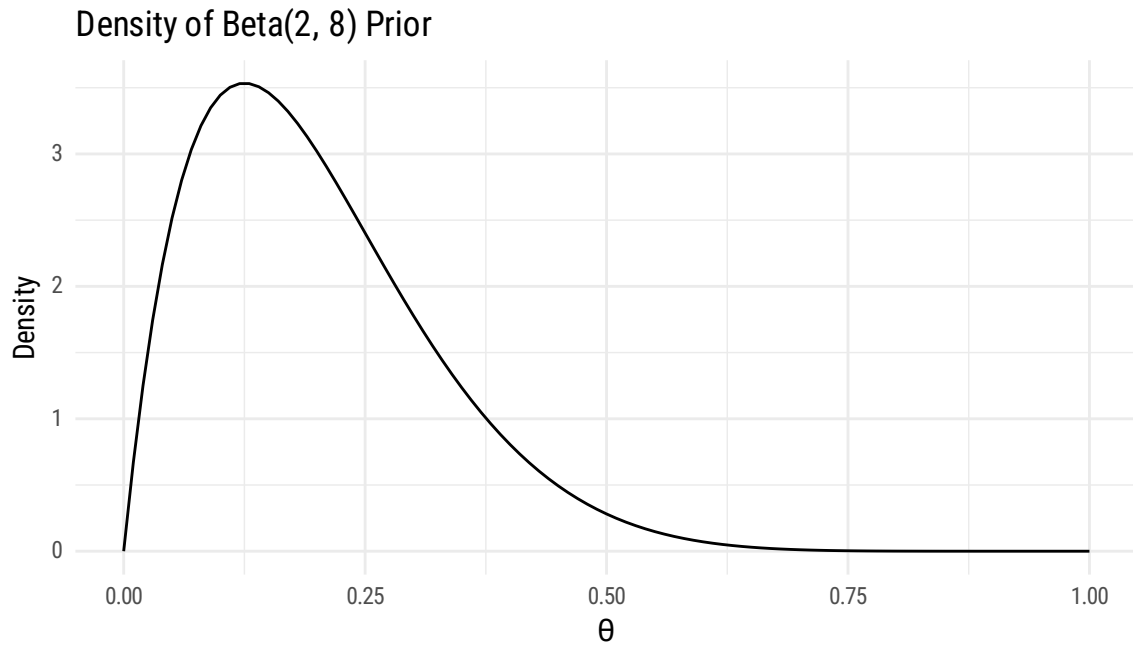
What is the prior probability that an individual in the group will experience at least 5 migraines?

From JAGS, the prior probability that an individual group will experience at least 5 migraines is 0.3547.

5.

Now suppose the expert provides the values 20% and 50% instead of 20% and 30%. Repeat 1 - 3 and contrast the results with what you obtained before.

Select a Beta prior to reflect the experts opinion and justify your choice.

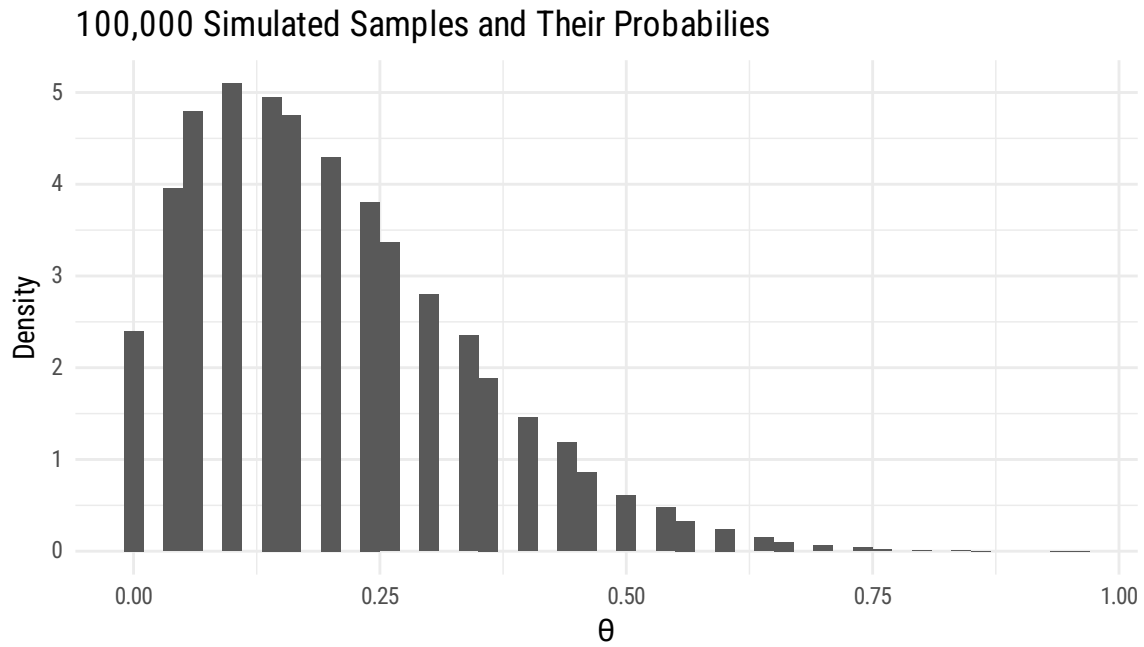


Using the guidelines on page 228 in the notes, I chose a prior by imposing $\mu = 0.20$ with a standard deviation of $\sigma = 0.15$. I felt this left room for values beyond the 20% and 30% elicited values, but focused the prior in the center of that. I used the following formulas to elicit shape values:

$$a = \frac{-\mu(\sigma^2 + \mu^2 - \mu)}{\sigma^2} \quad \text{and} \quad b = \frac{\mu - 1(\sigma^2 + \mu^2 - \mu)}{\sigma^2}.$$

These yielded the shape parameters $a = 1.2$ and $b = 4.8$. This would yield an effective sample size of $n = 6$, which feel appropriate. To get whole shape parameters, we went with $a = 2$ and $b = 8$ for an ESS of $n = 10$.

Use the prior predictive distribution to generate a sample that can be used as feedback to the expert. Provide the feedback graphically.



What is the ESS of the prior? Does this seem reasonable?

The ESS of the prior is $n = 10$. This seems far more reasonable

What is the prior probability that an individual in the group will experience at least 5 migraines?

From JAGS, the prior probability that an individual group will experience at least 5 migraines is 0.3621.