STA 6360, Report 3.4

Carson Slater *Baylor University*

Problem

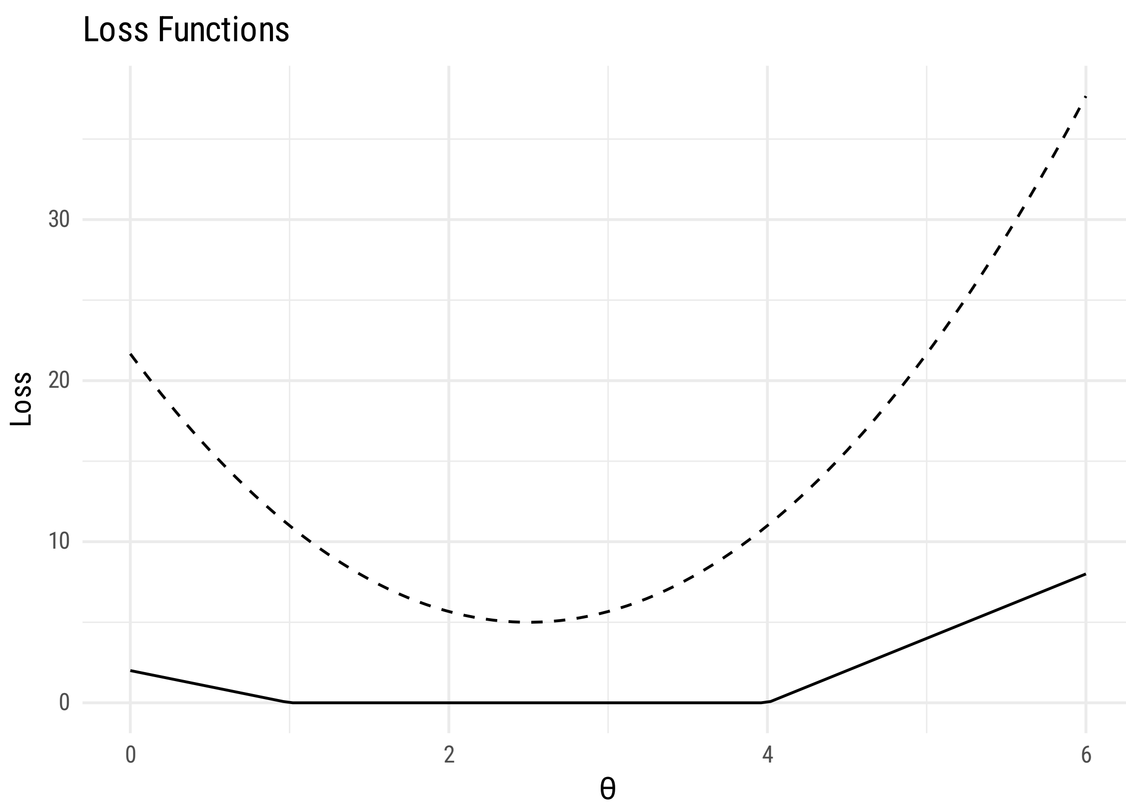
Suppose we wish to construct an interval estimate (a_1, a_2) for $\theta \in \mathbb{R}$.

1.

Using the following formula,

$$L(\theta, (a_1, a_2)) = \begin{cases} L_1(a_1 - \theta) & \text{if } \theta < a_1 \\ 0 & \text{if } a_1 \leq \theta \leq a_2 \\ L_2(\theta - a_2) & \text{if } a_2 < \theta. \end{cases}$$

suppose $L_1 = 2$ and $L_2 = 4$. For the interval with $a_1 = 1$ and $a_2 = 4$, plot both loss functions as functions of θ.



Loss funtion for 3.1.15 (solid) and 3.1.17 (dashed).

2

Now suppose $L_1 = 5$ and $L_2 = 10$. Let $\mathbf{x} = (x_1, \ldots, x_{20})$ be a vector of observations from a $\mathcal{N}(\theta, 1)$ distribution. Find the Bayes optimal interval estimates for θ under these two loss functions if $\theta \sim \mathcal{N}(0, 10^2)$ and $\bar{x} = 12$.

We know that via conjuagacy,

$$\mathbb{E}[\theta|\mathbf{x}] = \left(\frac{10^2}{10^2 + 1/20}\right) 12 + \left(\frac{1/20}{10^2 + 1/20}\right)(0) = 11.9940,$$
$$\mathbb{V}[\theta|\mathbf{x}] = \frac{(1/20)10^2}{10^2 + 1/20} = 0.04998.$$

We have that the interval minimizing the posterior expected loss is,

$$\mathbb{E}[\theta|\mathbf{x}] \pm \sqrt{\frac{L_1}{L_2}\mathbb{V}[\theta|\mathbf{x}]},$$

which yields the following interval,

$$11.9940 \pm \sqrt{\frac{5}{10}(0.04998)},$$

which is $(11.8360, 12.1521)$.